Measurement of Counting Statistics of Electron Transport in a Tunnel Junction

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(Received 15 April 2005; published 17 October 2005)

We present measurements of the time-dependent fluctuations of electrical current in a voltage-biased tunnel junction. We were able to simultaneously extract the first three moments of the current counting statistics. Detailed comparison of the second and the third moments reveals that the statistics is accurately described as Poissonian, expected for spontaneous current fluctuations due to electron charge discreteness, realized in tunneling transport at negligible coupling to environment.

DOI: 10.1103/PhysRevLett.95.176601 PACS numbers: 72.70.+m, 73.50.Td, 74.40.+k, 85.30.Mn

Photon counting statistics [1,2] is a key technique of quantum optics, which is routinely used to characterize the complexity of optical states, such as photon coherence, entanglement, and squeezing. In contrast, the subject of electron counting and noise statistics, which is expected to provide new insights into quantum transport, is essentially at infancy as an experimental field, with the first advances made only recently [3–6]. Electron counting proves to be far more challenging than that of photons primarily because of the extremely high frequency of electron passage events at typical current intensity, which requires fast charge detection. While in some cases Coulomb blockade can be used to localize electrons, bringing the tunneling rate down to the radio frequency range [4–6], the time resolution needed to measure free, nonlocalized electrons remains a challenge.

Another difficulty stems from the simple fact that, while photons do not interact, the electrons do. The electric field fluctuations produced by the electrons that are being measured can propagate out to the environment and perturb it. In turn, the environment noise, modulated by the signal, can couple back to the region of interest, strongly affecting the measured signal [3,7–11].

The electron problem is especially rich and intriguing: The electrons remain part of the many-body system while being detected, allowing quantum phenomena to manifest themselves in electric noise. This leads to a number of dramatic effects observed in electric noise, such as, notably, elementary charge transmutation in the fractional quantum Hall effect [12–14] and charge doubling in normal-superconducting junctions [15].

The regime in which electric fluctuations occur spontaneously due to charge discreteness, rather than due to thermal fluctuations, is realized at sufficiently low temperatures [16–18]. It was first demonstrated about 10 years ago in semiconductor point contacts [19,20] and in mesoscopic wires [21,22]. Current fluctuations in this regime are traditionally analyzed using the noise frequency spectrum. However, a more detailed description [23,24] is provided by the statistics $P(q)$ of the charge $q(\tau)$ transmitted through a conductor during a fixed time interval τ which, in principle, can be small or large compared to the time $\tau_0 = e/I$. The low frequency shot noise is just the second central moment of $P(q)$ at $\tau \gg \tau_0$:

$$S^{(2)} = \overline{|\delta I_\omega|^2}_{\omega=0} = \frac{\langle \Delta q^2(\tau)\rangle}{\tau} = \frac{\langle (q(\tau) - \bar{q})^2\rangle}{\tau} \quad (1)$$

with $\langle \cdots \rangle$ a shorthand for $\int \ldots P(q, \tau) dq$. The counting distribution $P(q)$, which encodes the full information about noise statistics, was studied in various regimes, including mesoscopic systems [23], NS junctions [25,26], and photon-assisted transport [27]. Specifics of the tunneling regime were considered in [28,29].

The present work extends the shot noise measurements beyond the second moment in a way uncorrupted by the presence of electromagnetic environment. In the low transmission tunnel junctions, used in our experiment, independent and uncorrelated charge transmission events, and thus Poissonian statistics, are expected. Our results confirm this expectation, paving the way to investigation of counting statistics. We measure current fluctuations by detecting the probability distribution of voltage across a load resistor, $P(V)$. The latter is directly related to $P(q)$ when the load resistance is made much smaller than the tunnel resistance (see below).

The knowledge of $P(q)$ allows one, in principle, to obtain all moments of q. However, with the measurement times $\tau \gg \tau_0$, due to the central limit theorem, the high moments become increasingly dominated by the lower order moments. This makes the *irreducible* parts of the moments, the cumulants, which contain new information, increasingly difficult to extract. Thus here we focus on the third central moment $\langle \Delta q^3\rangle = \langle (q - \bar{q})^3\rangle$ which coincides with the third cumulant $\langle\langle q^3\rangle\rangle$.

The expected statistics are different in the voltage-biased and current-biased regimes. The former case is described by the rate of attempts to pass equal to eV/h and a binomial distribution of successes [23]. At weak transmission, realized in our tunnel junction, the distribution assumes Poisson form, with the low frequency spectral density $S^{(k)}$ of the kth cumulant, corresponding to "long" measurement times $\tau \gg \tau_0$, expressed as

$$S^{(k)} = \frac{\langle\langle q^k \rangle\rangle}{\tau} = e^{k-1} I. \quad (2)$$

In a current-biased sample, in contrast, the rate of successes is fixed at I/e, with the attempts to pass, described by voltage fluctuations, characterized by Pascal distribution [8]. Theory of high-order noise statistics in the diffusive regime was developed in Ref. [7], the role of environment was considered in Ref. [9] (see also [10]).

In the first measurement of the third moment $S^{(3)}$ Reulet *et al.* [3] used a low impedance tunnel junction ($R_s \simeq 50\ \Omega$) as a noise source, with a parallel 50 Ω load impedance R_l of the cable used to feed the voltage to an external amplifier. The initial results [3] distinctly different from the Poissonian, in both magnitude and sign, suggested that $S^{(3)}$ is dominated by the effects extraneous to the junction. A theoretical investigation [8,9] clarified the importance of the environment for correct interpretation of the results, obtaining

$$S^{(3)} = S_s^{(3)} - 3R_{s,l}(S_s^{(2)} + S_l^{(2)}) \frac{\partial(S_s^{(2)} - S_l^{(2)})}{\partial V} \quad (3)$$

with $R_{s,l} = R_s R_l/(R_s + R_l)$ the impedance of the sample in parallel with the load, $S_s^{(2)}$ and $S_l^{(2)}$ the sample and load noise, and $S_s^{(3)}$ generated by the voltage-biased sample. The load resistor, being macroscopic, is not expected to produce a third cumulant [7,18]. The measurement [3], which due to $R_l \simeq R_s$ was neither fully in the voltage-biased nor in the current-biased regime, was well described by the last term of Eq. (3), dominated by the voltage-dependent $S_s^{(2)}$. Only at room temperature, due to low $\partial S_s^{(2)}/\partial V$, the result had the sign of $S_s^{(3)}$.

To measure the *intrinsic* $S^{(3)}$ free of the admixture of the second moment, we use a new method suggested and analyzed in Ref. [28] (Fig. 1). Current fluctuations generated by the sample (voltage-biased tunnel junction of high impedance) produce voltage fluctuations on the load resistor R_l, which are amplified and analyzed with computer. The statistics of voltage fluctuations on R_l, as discussed below [Eq. (6)], is identical to that of the intrinsic current fluctuations in the junction, provided that R_l is much smaller than the junction differential resistance and the voltage drop across the junction is bigger then the temperature [8].

The main source of errors in the measurement of the kth cumulant $\langle\langle q^k \rangle\rangle$ of the distribution $P(q)$ is statistical. In order to estimate the signal-to-noise ratio, the measured value $\langle\langle q^k \rangle\rangle$ should be compared to its variance $\mathrm{var}(\Delta q^k)$ due to both sample and amplifier noise. The variance is expressed through the central moments of the order $2k$. The variance of an *even* order for a generic distribution can be estimated with the help of the central limit theorem, using Gaussian statistics:

$$\mathrm{var}\,(\Delta q^k) = (\langle \Delta q^{2k} \rangle)^{1/2} \simeq ((2k-1)!!)^{1/2} \langle \Delta q^2 \rangle^{k/2}. \quad (4)$$

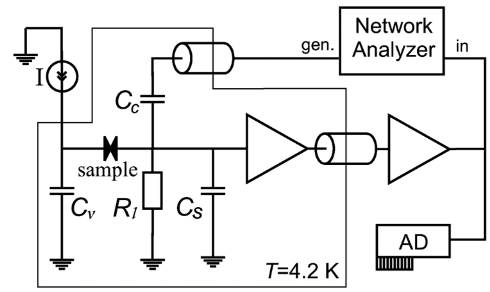

FIG. 1. Schematic of the experimental setup. The current source drives constant *average* current I while the capacitor C_v fixes the voltage across the sample and the load at relevant frequencies. The bandwidth is determined by the load resistance R_l and stray capacitance C_s. The voltage fluctuations across the load resistor are amplified and digitized by a 12 bit analog-to-digital (A/D) convertor (Ultraview Corp.) with 5 ns sampling time and 20 ns interval between the samples. The amplifier input impedance is determined by R_l.

Here the variance $\langle \Delta q^2 \rangle$ in Eq. (4) is due to the sample, amplifier, and load noise, $S^{(2)} = S_s^{(2)} + S_l^{(2)} + S_a^{(2)}$, assuming all three to be uncorrelated. The signal-to-noise ratio for a single measurement of the third cumulant is thus estimated as a ratio of $S_3\tau$ to $15^{1/2}(S_2\tau)^{3/2}$, and therefore it is beneficial to reduce the sampling time τ to improve sensitivity, in accord with the central limit theorem. However, the amplified signal is correlated at short times due to the finite bandwidth of the input circuit. This makes the effective sampling time restricted from below by the sample parasitic capacitance, C_s, of both intrinsic and stray kind, and by the effective input resistance $R_{s,l}$, so that $\tau_{\mathrm{eff}} \simeq R_{s,l} C_s$. Repeating the measurement N times would further reduce fluctuations by a factor $\sqrt{N}$, assuming statistically independent successive measurements. However, since the measurements separated in time by less than τ_{eff} are correlated, the maximal improvement is of the order of $\sqrt{\mathcal{T}/\tau_{\mathrm{eff}}}$, where $\mathcal{T}$ is the measurement time. In the case of a high impedance tunnel junction, the noise is dominated by the resistor R_l thermal noise, $2k_B T/R_l$. The measurements were performed at 4.2 K to reduce both this noise and the noise of the amplifier. We used a cryogenic amplifier built with ATF-35143 transistors. The amplifier voltage noise, expressed as $2k_B TR$, translates into $R \lesssim 500\ \Omega$ at the bath temperature. Ignoring for now both the shot noise produced by the sample and the amplifier noise, we estimate the signal-to-noise ratio as

$$S/N = \frac{e^2 I \sqrt{\mathcal{T}}}{\sqrt{15}(2k_B T/R_l)^{3/2}\sqrt{\tau\tau_{\mathrm{eff}}}}. \quad (5)$$

Replacing τ by τ_{eff} and plugging in Eq. (5) we find the signal-to-noise ratio $S/N \propto R_l^{1/2} C_s^{-1}$. It is therefore clear that it pays to decrease C_s and increase R_l to improve S/N. We placed a cryogenic amplifier in the vicinity of the sample to reduce as much as possible the capacitance C_s. The choice of the R_l is restricted by the desire to keep most of the signal at frequencies well above the $1/f$ corner of the amplifier. We chose $R_l = 9.1\ \mathrm{k}\Omega$, which together with the stray capacitance of about 4 pF gives $\tau_{\rm eff} = R_l C_s \simeq$ 40 ns and signal bandwidth $(2\pi\tau_{\rm eff})^{-1} \approx 4$ MHz. We used a current source to set an *average* current. To preserve the voltage bias conditions, a capacitor C_v was introduced, which kept the voltage constant across the sample-load circuit at all relevant frequencies.

We used tunneling through a 30 Å thick SiO_2 gate oxide of a p-channel Si field-effect transistor to produce a shot noise. In this system tunneling occurs only under negative gate voltage required to induce a hole channel. The differential resistance of the barrier $R_s = (\partial I/\partial V)^{-1} > 10^7\ \Omega$ was much bigger than R_l, placing the sample securely in the voltage-biased regime. Indeed, the maximal contribution of the second term in Eq. (3),

$$-3Z(eI + 2k_BT/R_l)e\frac{\partial I}{\partial V} \approx -\frac{6ek_BT}{(R_l + R_s)}, \qquad (6)$$

is estimated as $5.2 \times 10^{-48}\ \mathrm{A}^3/\mathrm{Hz}^2$, which is 2 orders of magnitude smaller than $e^2 I \sim 5 \times 10^{-46}\ \mathrm{A}^3/\mathrm{Hz}^2$ (Fig. 2 and 3).

The amplified voltage V_a was sampled with an A/D convertor during 5 ns intervals about 10^{10} times at each bias current value. Typical distribution $P(V_a)$ is presented in Fig. 2, upper inset. To clean the obtained histogram, we correct for nonequal bin widths of the converter by calibrating the latter against a linearly swept signal (Fig. 3, lower inset). The effect of normalizing $P(V_a)$ with the converter calibration, illustrated in Fig. 2 lower inset, is twofold. First, the histogram loses the noisy features on the smallest scale. Simultaneously, the envelope is somewhat corrected on a large scale. The latter effect, due to averaging, has longer stability time, which is fortunate, since the noisy features, while less stable, were found to have little effect on the second and third cumulants.

For a linear circuit, the amplified voltage $V_a(\omega)$ and the input current $i(\omega)$ Fourier components are proportional: $V_a(\omega) = Z(\omega)K(\omega)i(\omega)$, with Z the load impedance and K the amplification. Therefore the third cumulant is related to the respective spectral densities of V_a as

$$\langle V_a^{(3)}\rangle = \frac{S^{(3)}}{(2\pi)^2}\iint_{-\infty}^{+\infty} A(\omega)A(\omega')A(-\omega-\omega')d\omega d\omega' \quad (7)$$

with $A(\omega) = Z(\omega)K(\omega)$. For the broadband amplifiers used, the amplification $K(\omega)$ was almost frequency independent between the low $\sim$0.5 MHz and high $\sim$25 MHz frequency cutoffs intentionally introduced to filter the $1/f$ amplifier noise and the wide-band noise at frequencies well above $(2\pi\tau_{\rm eff})^{-1}$. The complex product $Z(\omega)K(\omega)$ was obtained from calibration, introducing a known current through a small capacitor C_c typically of 2.4 pF. We then used Eq. (7) for $V_a^{(3)}$ and a similar expression for $V_a^{(2)}$ to extract $S^{(3)}$ and $S^{(2)}$. The second cumulant $S^{(2)}$ combines the shot, thermal, and amplifier noise contributions. Since only the former depends on current, the amplifier noise and the thermal noise, obtained from $S^{(2)}$ at zero current, can easily be subtracted [30]. The resulting noise $S^{(2)}$, shown in Fig. 2, varies linearly with current as expected ($eV \simeq 4$ eV $\gg k_BT$ at all currents). The measured value agrees with the expected value, eI, within a calibration error of 5%.

In order to extract properly the third cumulant of the voltage, one should account for the effect of amplification nonlinearity that can mix $S^{(2)}$ with $S^{(3)}$. Let us consider the amplifier nonlinearity, $V_{\rm out} = K(V_{\rm in} + V_{\rm in}^2/U)$, which contributes to $\langle \delta V_{\rm out}^3\rangle$ as follows:

$$\langle \delta V_{\rm out}^3\rangle/K^3 - \langle \delta V_{\rm in}^3\rangle = 9\langle \delta V_{\rm in}^2\rangle^2/U + O(U^{-2}). \qquad (8)$$

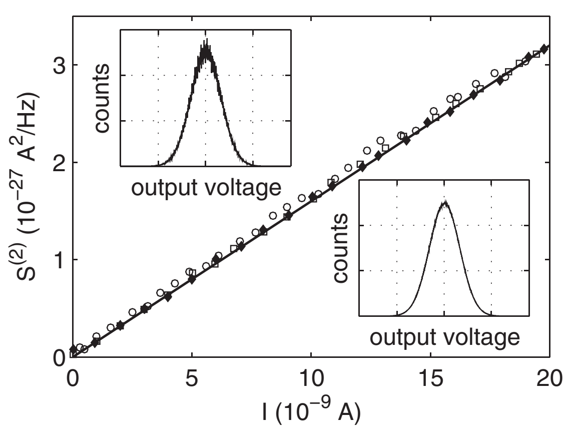

FIG. 2. Measured value of the noise, $S^{(2)}$, as a function of the current. Different markers represent the data of three different measurements. The straight line $S^{(2)} = eI$, corresponding to the expected noise intensity [32], is shown for guidance. Upper inset: raw voltage histogram sample. Lower inset: cleaned histogram, illustrating the result of normalizing with the A/D converter calibration (see text).

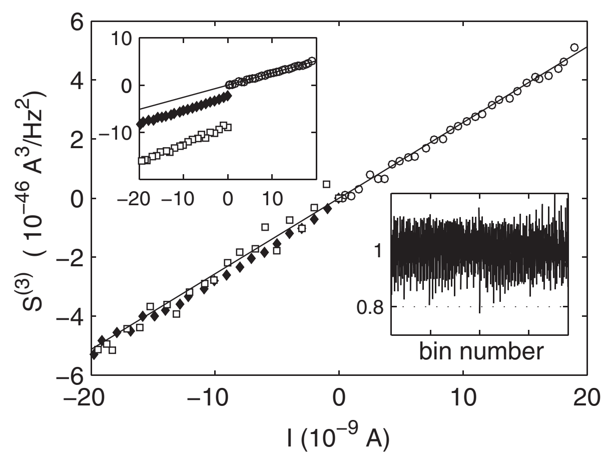

FIG. 3. Measured third cumulant $S^{(3)}$ of the transmitted charge, obtained separately for different current directions. (Markers are the same as in Fig. 2; the straight line is $S^{(3)} = e^2 I$.) Upper inset: $S^{(3)}$ vs I without amplifier nonlinearity correction. Lower inset: normalized histogram of the linearly swept signal, used to calibrate the A/D converter (see text).

The right-hand side would affect the value $S^{(3)}$ calculated from $\langle \delta V_{\rm out}^3 \rangle$ (similar analysis for $\langle \delta V_{\rm out}^2 \rangle$ yields $\langle \delta V_{\rm out}^2 \rangle / K^2 - \langle \delta V_{\rm in}^2 \rangle \approx 2\langle \delta V_{\rm in}^3 \rangle / U$). The parameter U can be estimated by applying an ac signal and measuring its second harmonic. We took special care to reduce nonlinearity of the amplifier track, especially the last amplifier in the chain. We managed to make it comparable to the nonlinearity of the cryogenic amplifier, which was different for different measurements, depending on the regime. We found U to exceed 20 mV, thus having a negligible effect on $S^{(2)}$. However, estimates show that the nonlinearity correction to $S^{(3)}$ can be as large as 10^{-45} A^3/Hz^2. We attribute the third cumulant at zero current (see the upper inset of Fig. 3) to this nonlinearity and determine U as the ratio $9\langle \delta V_{\rm in}^2 \rangle_{I=0}^2 / \langle \delta V_{\rm out}^3 \rangle_{I=0}$. Using this U, we subtract the nonlinearity contribution, Eq. (8), from the data at all currents. This procedure, as illustrated in Fig. 3, upper inset, restores zero value at $I = 0$, but has little effect on the slope of the dependence $S^{(3)}$ vs I. We therefore conclude that the nonlinearity, while necessary to account for to improve accuracy, is not strong enough to compromise the measurement of $\partial S^{(3)}/\partial I$. The results, shown in Fig. 3, are found to be in excellent agreement with the Poissonian $S^{(3)} = e^2 I$.

As an additional check, we also reversed the current direction. Since we had to apply negative voltage on the gate to induce holes, it required rebonding the sample (see Fig. 3, upper inset). We found the third cumulant to be an odd function of current, as expected.

Finally, we note that the literature is not entirely unanimous regarding the Poissonian character of tunneling transport. Dissent is exemplified by Ref. [31], predicting a new "quantum regime" at sufficiently low frequencies, limited by inverse flight time in a noninteracting fermion model. Reference [31] obtains $S^{(3)}$ for the tunneling current of the sign opposite to Poissonian $S^{(3)}$ and also of much smaller magnitude, quadratic in transmission rather than linear as in Eq. (2) above. The conditions stated in Ref. [31] are fulfilled in our experiment: the time interval between electron tunneling and its detection, estimated from EM signal propagation speed, is of order 3×10^{-11} s, which is much shorter than the sampling time, 5 ns, placing the measurement securely at low frequency in the sense of Ref. [31]. The results [31] are thus not in accordance with our observations.

In summary, we present the first measurements of the charge counting statistics in *voltage-biased* tunnel junction up to the third cumulant. The results, obtained by analyzing the distribution of transmitted charge, are in excellent agreement with expectations for the Poissonian process, making electron counting statistics amenable to experimental investigation.

We are indebted to O. Prus for his contribution at the early stage of the work. One of us (M. R.) is indebted to G. Lesovik for attracting his attention to the problem. This research was supported by the U.S.-Israel Binational Science Foundation and (D. S. and Yu. B.) by the Lady Davis grant.

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