ISSN 0021-3640, JETP Letters, 2013, Vol. 97, No. 3, pp. 149–154. © Pleiades Publishing, Inc., 2013.
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Observation of Crossover from Weak Localization to Antilocalization in the Temperature Dependence of the Resistance of a Two-Dimensional System with Spin–Orbit Interaction

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Received November 9, 2012; in final form, January 9, 2013

A nonmonotonic temperature dependence of the resistance with a maximum in the temperature range of 2–4 K whose position depends on the hole density has been observed in hole channels of silicon field-effect transistors. The spin–orbit hole relaxation time and the temperature dependences of the phase relaxation time of the electron wave have been obtained from the measurements of the alternating sign anomalous magnetoresistance. The nonmonotonic temperature dependence of the resistance can be described by the formulas of weak-localization theory with these parameters. The maximum appears owing to a temperature-induced change in the relation between the measured times. As a result, the localization behavior of the conductivity at high temperatures is changed to the antilocalization behavior at low temperatures. The inclusion of quantum corrections to the conductivity caused by the electron–electron interaction improves quantitative agreement between the experiment and calculation. Thus, it has been demonstrated that, in contrast to the widely accepted concept, there is a region of the parameters where the electron–electron interaction does not change the antilocalization (metallic) type of the temperature dependence of the resistance.

DOI: 10.1134/S002136401303003X

The spin–orbit interaction triggering the spin relaxation mechanism significantly modifies quantum corrections to the conductivity caused by localization effects [1] and the electron–electron interaction effects [2, 3] in disordered systems. In the weak localization theory, where the physical mechanism is the interference of electron waves at their scattering on impurities, the sign of the correction to the conductivity depends on the relation between the phase relaxation time of the electron wave τ_ϕ and the spin relaxation time owing to the spin–orbit interaction τ_{so} (in this work, we discuss only two-dimensional systems of charge carriers). At $\tau_{so} \gg \tau_\phi$, spin relaxation is insignificant and the temperature dependence of the resistivity $\rho(T)$ is determined by the localization of carriers and has the insulating form. In the opposite limit $\tau_{so} \ll \tau_\phi$, quantum correction to the conductivity of noninteracting electrons is positive (the so-called antilocalization effect). Since the spin relaxation time is independent of the temperature and the phase relaxation time is determined by inelastic processes, in the absence of other mechanisms of the temperature dependence of the resistance in the system with the spin–orbit interaction, an increase in $\tau_\phi(T)$ with a decrease in the temperature could lead (see, e.g., review [4]) to a nonmonotonic dependence $\rho(T)$ with the maximum at $\tau_\phi(T) \sim \tau_{so}$. We certainly observed this behavior of the resistance. The use of the times $\tau_\phi(T)$ and τ_{so} determined from anomalous magnetoresistance curves made it possible to satisfactorily describe the observed temperature dependence by the formulas of the weak-localization theory. At first glance, the obtained result contradicts the theory of quantum corrections taking into account the electron–electron interaction. This theory predicts the universal insulating behavior for systems with strong spin–orbit interaction (see Fig. 2 in [2] and Fig. 41 in review [3]). This behavior was confirmed in many investigations of thin metal films (see, e.g., [5]) and in hole channels of silicon field-effect transitions on the Si(111) surface [6]. The effect of the spin–orbit interaction on the electron–electron interaction is characterized by the parameter $T\tau_{so}/\hbar$. The indicated contradiction is likely explained by the fact that the temperature T_m at which the resistance is maximal corresponds to the transient regime $T_m\tau_{so}/\hbar \gtrapprox 1$, whereas the prediction of the universal insulating behavior was made in [2] under the condition $\tau_{so} \ll \tau_\phi \ll \hbar/T$.

The nonmonotonic temperature dependences of the resistance with the maximum were observed in two-dimensional systems with a high mobility of charge carriers in the transition region in the electron density from the insulating to the metallic state both in systems with weak spin–orbit interaction [7] and in systems where noticeable spin–orbit interaction could be expected [8, 9]. The nonmonotonic temperature

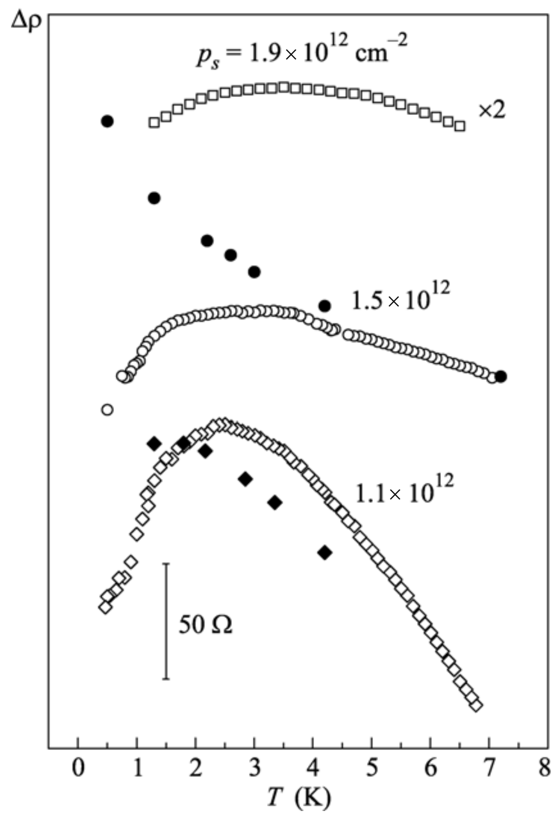

Fig. 1. Temperature dependences of change in the resistivity (per square) $\Delta\rho$ in (open symbols) zero magnetic field for three hole surface densities p_s. The resistivities at $T =$ 4.2 K were 2477, 3302, and 4902 Ω per square for $p_s = 1.9 \times 10^{12}$, 1.5×10^{12}, and 1.1×10^{12} cm^{-2}, respectively. The closed symbols depict the temperature dependences of the resistivity in the field $H = 0.2$ and 0.4 T for $p_s = 1.5 \times 10^{12}$ and 1.1×10^{12} cm^{-2}, respectively.

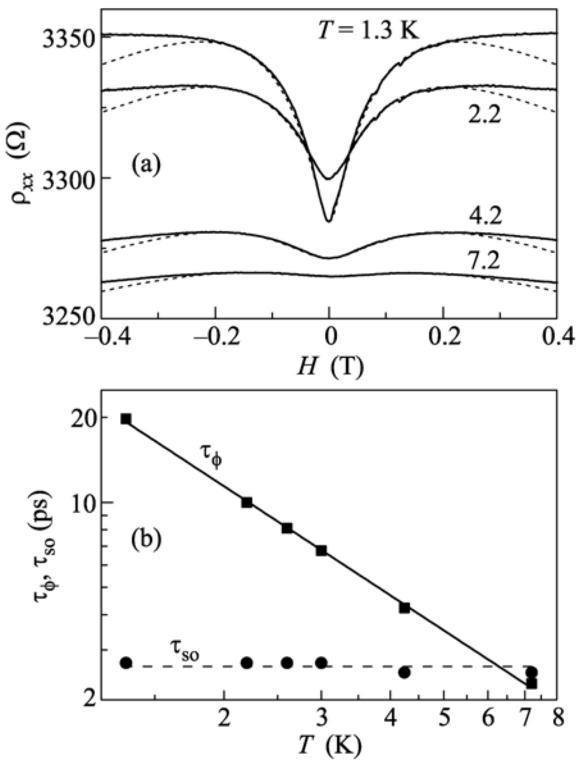

Fig. 2. (a) (Solid lines) Experimental magnetic field dependences of the resistivity of the sample ρ_{xx} for $p_s = 1.5 \times 10^{12}$ cm^{-2} at various temperatures. The line for $T =$ 4.2 K is shifted down by 26.6 Ω in order to avoid its intersection with other lines. The dashed lines are the calculation of the magnetoresistance by Eq. (1) with the spin relaxation times τ_{so} and phase relaxation times τ_ϕ shown in panel (b) by circles and squares, respectively. The solid straight line in panel (b) corresponds to the power-law temperature dependence $\tau_\phi = 27.9T^{-p}$ ps with $p = 1.29$ and the dashed straight line in panel (b) corresponds to the value $\tau_{so} = 2.62$ ps. These straight lines are the least squares fits of the respective experimental points.

dependence of the resistance with the maximum in systems with weak spin–orbit interaction can appear [10, 11] owing to the renormalization (dependences on the temperature and disorder) of the Fermi liquid parameter describing the electron–electron interaction. However, such a renormalization is usually important in systems with a low carrier density, particularly in the case of the presence of valley degeneracy, and is hardly significant in our samples. The nonmonotonic temperature dependence of the resistance under the conditions most corresponding to our experiment was observed on hole channels in multilayer Ge/SiGe heterostructures [12]. However, the origin of the maximum in the temperature dependence of the resistance was beyond the scope of works [8, 9, 12]. Remarkable results were obtained in [13], where the metallic temperature dependence of the conductivity was observed in hole channels of GaAs/InGaAs/GaAs heterostructures, whereas crossover was likely masked by the electron–phonon scattering.

The results of this work were obtained for accumulation hole channels of silicon field-effect transistors fabricated on the Si(110) surface. The hole mobility depended on their density and was about 10^3 cm^2/(V s) in the density range under study. Most of the experiments were performed in a cryostat with the evacuation of ^{4}He vapor in the temperature range of 1.3–7 K. Some measurements were carried out in a cryostat with the evacuation of ^{3}He vapor in the temperature range of 0.5–7 K. Anomalous magnetoresistance, Hall resistance, and Shubnikov–de Haas oscillations were measured in magnetic fields created by a superconducting solenoid. We studied two samples made of one silicon wafer. The results were the same for both samples. The hole density in the samples was determined from the period of Shubnikov–de Haas oscillations, as well as from the magnetic-field dependence of the Hall resistance.

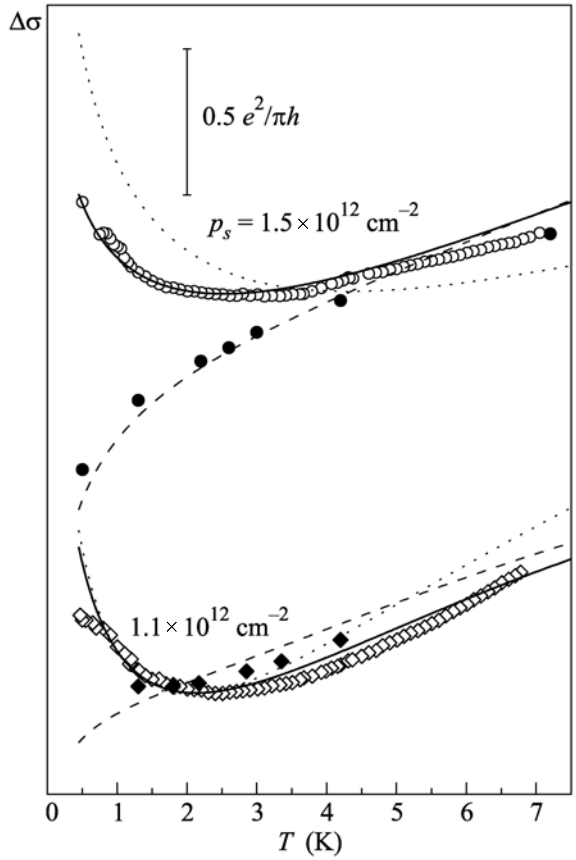

Fig. 3. (Points) Experimental temperature dependences of the conductivity recalculated from the data shown in Fig. 1 and the corresponding theoretical curves, which were obtained with the times τ_{so} and τ_ϕ determined from the measured anomalous magnetoresistance (the corresponding values for $p_s = 1.5 \times 10^{12}$ cm^{-2} are given in the caption of Fig. 2, and $\tau_{so} = 4.5$ ps and $\tau_\phi = 20.3T^{-1.46}$ ps for $p_s = 1.1 \times 10^{12}$ cm^{-2}). The dotted lines calculated by Eq. (2) are the temperature dependences of the conductivity in zero magnetic field determined by the weak localization effects. The solid lines were calculated as additive contributions from weak localization (Eq. (2)) and from the electron–electron interaction (Eqs. (3) and (4) with the interaction parameters given in the main text). The dashed lines are the calculated corrections to the conductivity in magnetic fields of 0.2 and 0.4 T for $p_s = 1.5 \times 10^{12}$ and 1.1×10^{12} cm^{-2}, respectively.

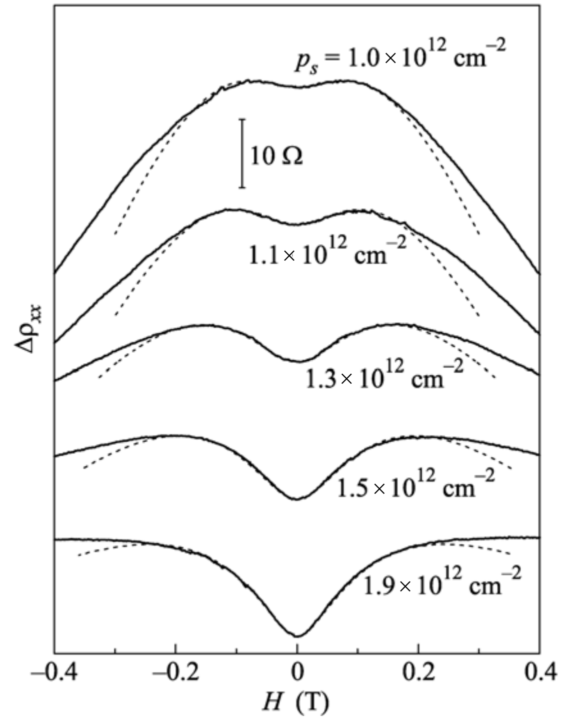

Fig. 4. Evolution of the anomalous magnetoresistance with variation of the hole density. The approximations of the experimental dependences by Eq. (1) are shown by dashed lines ($T = 4.2$ K).

Figure 1 shows the temperature dependence of the resistivity of a sample in zero and classically weak magnetic fields for three different hole densities p_s. All curves measured in zero field have a maximum, which shifts toward lower temperatures with a decrease in p_s. In magnetic field, the temperature dependence of the resistivity remains monotonic and insulating throughout the entire temperature range. Theories of quantum corrections should obviously be used to explain such effects. To reveal the role of weak localization effects, we measured the anomalous magnetoresistance observed in classically weak magnetic fields. Figure 2 shows the typical results of such measurements for the hole density $p_s = 1.5 \times 10^{12}$ cm^{-2} at various temperatures. As can be seen in Fig. 2, a weak ($H \gtrsim 0.1$ T) magnetic field suppresses the antilocalization correction. As a result, the temperature dependence becomes insulating. The observed nonmonotonic magnetic field dependence of the resistance is characteristic of the weak-localization effects in the system with a quite fast spin–orbit relaxation. To quantitatively describe the anomalous magnetoresistance curves, we used the Hikami–Larkin–Nagaoka formula [1]

$$\delta\sigma(H) - \delta\sigma(0) = \frac{e^2}{2\pi^2\hbar}\left[\psi\left(\frac{1}{2} + \frac{H_\phi + H_{so}}{H}\right) + \frac{1}{2}\psi\left(\frac{1}{2} + \frac{H_\phi + 2H_{so}}{H}\right) - \frac{1}{2}\psi\left(\frac{1}{2} + \frac{H_\phi}{H}\right) - \ln\frac{H_\phi + H_{so}}{H} - \frac{1}{2}\ln\frac{H_\phi + 2H_{so}}{H} + \frac{1}{2}\ln\frac{H_\phi}{H}\right]. \quad (1)$$

Here, $\delta\sigma$ is the quantum correction to the conductivity of a two-dimensional system; ψ is the digamma function; and $H_x = \dfrac{c\hbar}{4eD\tau_x}$, where D is the hole diffusion coefficient and x is one of the used subscripts. For small changes in the magnetoresistance, $\rho_{xx}(H) - \rho(H=0) \approx -\rho^2(H=0)[\delta\sigma(H) - \delta\sigma(0)]$.

We used Eq. (1) to describe the anomalous magnetoresistance in our samples because of the results reported in [14], where it was shown that this formula remains valid for systems with the spin–orbit splitting of the spectrum proportional to the third power of the wave vector of two-dimensional carriers whose spin relaxation occurs through the D'yakonov–Perel' mechanism [15], which is most efficient in semiconductors. Theoretical calculations performed for two-dimensional hole systems within the effective mass approximation predict the cubic wave-vector dependence of the spin–orbit splitting associated with the asymmetry of the potential well in which two-dimensional holes are located (see, e.g., review [16] and references therein). However, our choice is not the only possible. The spin–orbit relaxation in hole systems can also occur in the absence of the spin–orbit splitting of the two-dimensional spectrum owing to the elastic scattering between the states of light and heavy holes mixed because of the quantum confinement effect. This was demonstrated in [17] and was used to describe the experiment on the anomalous magnetoresistance in hole channels of GaAs/AlGaAs heterostructures in [18]. A reason for our choice is the observation of beats of Shubnikov–de Haas oscillations in our samples (for more details of this effect, see [19]), which certainly indicates that spin degeneracy is lifted in this system owing to the spin–orbit interaction.

Accurate independent determination of the spin and phase relaxation times from the experimental curves is possible only for a nonmonotonic magnetic field dependence of the magnetoresistance. This circumstance limits the hole density values for which such a procedure can be performed (see Fig. 4). The τ_{so} and τ_ϕ values determine the position and height of the maximum of the magnetoresistance in the magnetic field, respectively. We approximated the experimental curves by the calculated lines trying to reproduce the position and height of the maximum and obtained a satisfactory agreement for reasonable τ_{so} and τ_ϕ values. In particular, in agreement with the commonly accepted point of view, the spin–orbit relaxation time is independent of the temperature, whereas the phase relaxation time increases with a decrease in the temperature as $\tau_\phi \propto T^{-p}$ (see Fig. 2b). Deviations of the calculated magnetoresistance from the experimental data in strong magnetic fields observed in Fig. 2a can be attributed to various reasons. First, Eq. (1) is applicable only in the so-called diffusion approximation when $H < H_{tr} = \dfrac{c\hbar}{4eD\tau}$, where τ is the carrier momentum relaxation time. Under the experimental conditions, H_{tr} decreases from 0.77 to 0.41 T with an increase in the hole density from $n_s = 0.98 \times 10^{12}$ cm^{-2} to $n_s = 1.68 \times 10^{12}$ cm^{-2}, respectively. Deviations from this formula are manifested even at $H < H_{tr}$ (see, e.g., [20] and references therein). Second, there is a classical temperature-dependent mechanism of the magnetoresistance for a system consisting of two groups of carriers (in our case, these are holes corresponding to two branches of the spectrum) that was successfully used to explain the temperature dependence of the magnetoresistance in two-dimensional hole channels of GaAs/AlGaAs heterostructures [21]. For weak magnetic fields, this mechanism makes a positive contribution to the magnetoresistance proportional to the magnetic field squared. Thus, the difference between the experimental and theoretical curves in stronger fields can be attributed to this mechanism.

Determining the time τ_{so} and interpolating the experimental dependence $\tau_\phi(T)$ by the power-law temperature dependence, we can try to describe the experimental temperature dependences of the resistance. However, it is more convenient to consider the conductivity σ, which is inversely proportional to the resistivity. Figure 3 shows the experimental data and theoretical results obtained with the weak localization theory [14] for the conductivity (we note that the sign in the corresponding formula in [14] should be changed to the opposite):

$$\delta\sigma(H=0) = \frac{e^2}{2\pi^2\hbar}\left[-\frac{1}{2}\ln\frac{\tau}{\tau_\phi} + \ln\left(\frac{\tau}{\tau_\phi} + \frac{\tau}{\tau_{so}}\right) + \frac{1}{2}\ln\left(\frac{\tau}{\tau_\phi} + \frac{2\tau}{\tau_{so}}\right)\right]. \quad (2)$$

This equation provides the minimum in correction to the conductivity at $\tau_\phi = \tau_{so}(\sqrt{5} + 1)/2 \approx 1.62\tau_{so}$. The dotted lines calculated by Eq. (2) with the parameters τ_{so} and dependences $\tau_\phi(T)$ obtained from the measurements of the anomalous magnetoresistance for the corresponding hole densities reproduce the qualitative behavior and scale of change in the conductivity with the temperature. It is noteworthy that the relations $T\tau_{so}/\hbar \approx 1$ and 1.3 at $p_s = 1.5 \times 10^{12}$ and 1.1×10^{12} cm^{-2}, respectively, are valid for temperatures T_m at which the maxima of the resistance are observed in the experiment. For this reason, it can be assumed that the spin–orbit interaction under our conditions hardly affects the electron–electron interaction. We numerically calculated corrections from the electron–electron interaction using formulas from [22], where the interaction was described in all regimes, including the previously considered diffusive ($T\tau/\hbar \ll 1$) [3] and ballistic ($T\tau/\hbar \gg 1$). These calculations show that the observed nonmonotonic temperature dependence cannot be reproduced if it is assumed, according to [2], that the triplet term in the diffusion regime is suppressed by the spin–orbit interaction. However, the triplet term can improve agreement with the experiment, as will be shown below. The significance of the triplet term at a small parameter $T\tau_{so}/\hbar \lesssim 0.05$ was mentioned in recent experimental work [23] specially devoted to this problem. For these reasons, we used equations from [22] including both the singlet,

$$\delta\sigma_C = \frac{e^2}{\pi\hbar}\frac{T\tau}{\hbar}\left[1 - \frac{3}{8}f(T\tau/\hbar)\right] - \frac{e^2}{2\pi^2\hbar}\ln\frac{E_F}{T}, \quad (3)$$

and triplet,

$$\delta\sigma_T = \frac{e^2}{\pi\hbar}\frac{T\tau}{\hbar}\frac{3\tilde{F}_0^\sigma}{1+\tilde{F}_0^\sigma}\left[1 - \frac{3}{8}t(T\tau/\hbar;\, \tilde{F}_0^\sigma)\right] - \frac{3e^2}{2\pi^2\hbar}\left[1 - \frac{1}{F_0^\sigma}\ln(1+F_0^\sigma)\right]\ln\frac{E_F}{T}, \quad (4)$$

terms. Here, E_F is the Fermi energy of the two-dimensional system. The functions f and t in Eqs. (3) and (4), respectively, are cumbersome and specified by Eqs. (3.36) and (3.44) in [22], respectively. The Fermi liquid parameters F_0^σ and $\tilde{F}_0^\sigma$ are determined by the dimensionless interaction parameter that is the Coulomb-to-kinetic-energy ratio $r_s = \sqrt{2}\,e^2/(\hbar\epsilon V_F)$, where V_F is the Fermi velocity of two-dimensional charge carriers and ϵ is the dielectric constant of the medium surrounding the two-dimensional system (r_s was calculated with the dielectric constant of silicon ϵ = 11.5). In the hole density range used in the experiment, r_s is between 2.7 and 3.8. The Fermi liquid parameters F_0^σ and $\tilde{F}_0^\sigma$ are given by the expressions [22]

$$\tilde{F}_0^\sigma = -\frac{1}{2}\frac{r_s}{r_s + \sqrt{2}}, \quad (5)$$

$$F_0^\sigma = -\frac{1}{\pi}\frac{r_s}{\sqrt{r_s^2 - 2}}\arctan\sqrt{r_s^2/2 - 1}, \quad r_s^2 > 2. \quad (6)$$

Using the parameters $F_0^\sigma = -0.41$ and $\tilde{F}_0^\sigma = -0.36$ calculated by these formulas with the hole density p_s = 1.1×10^{12} cm^{-2}, we obtain good agreement with the experiment (the lower solid line in Fig. 3) without the fitting parameters (the only fit is the vertical displacement of the curves up to coincidence of absolute values of the conductivity at the minimum). To describe the dependence at $p_s = 1.5 \times 10^{12}$ cm^{-2} (r_s = 3.0), we had to use the parameters $F_0^\sigma = -0.35$ and $\tilde{F}_0^\sigma = -0.29$ corresponding to r_s = 2.0. We did not try to describe the nonmonotonic temperature dependence at p_s = 1.9×10^{12} cm^{-2}, because it is impossible to obtain reliable values τ_{so} and τ_ϕ at this density in view of the absence of a pronounced maximum of the anomalous magnetoresistance (see Fig. 4). An increase in the hole density is accompanied by an increase in the positive contribution to the magnetoresistance primarily owing to a decrease in the spin relaxation time.

It is worth noting that the theory of quantum corrections [22] owing to the electron–electron interaction predicts the nonmonotonic temperature dependence of the resistance near the parameter value $r_s \sim$ 3.5. However, this dependence has a minimum. The appearance of the minimum is due to opposite signs of the contributions to the conductivity from the ballistic (proportional to the temperature) and diffusive (proportional to the logarithm of the temperature) terms. It is important that the position of the minimum in the temperature depends both on the transport time τ and on the interaction parameters. For example, the extremum at $F_0^\sigma = \tilde{F}_0^\sigma = -0.4$ is reached at $T\tau/\hbar \approx 0.03$ (see Fig. 8 in [22]), i.e., seemingly in the pure diffusive regime. The maxima in the temperature dependence of the conductivity calculated by Eqs. (3) and (4) are at $T \approx 1.6$ and 23 K for $p_s = 1.1 \times 10^{12}$ and 1.5×10^{12} cm^{-2}, respectively. This explains the opposite signs and significantly different magnitudes of the calculated shifts of the minimum of the conductivity owing to corrections from the interaction for different hole densities in Fig. 3.

To conclude, we have shown that the mechanism of the observed crossover in the temperature dependence of the resistivity is the transition from weak localization to antilocalization caused by a change in the relation between the spin–orbit and phase relaxation times. Our analysis indicates that there is a region of parameters where antilocalization is manifested in the temperature dependence of the resistivity for two-dimensional systems with spin–orbit interaction, in contrast to the commonly accepted opinion that the behavior of such systems is universally insulating, as was predicted by Altshuler and Aronov [2], considering the quantum corrections associated with the electron–electron interaction.

We are grateful to L.E. Golub, G.M. Min'kov, I.L. Aleiner, A.V. Germanenko, P.M. Ostrovskii, V.F. Gantmakher, and V.T. Dolgopolov for stimulating discussions. This work was supported in part by the Division of Physical Sciences, Russian Academy of Sciences (program "Spin Phenomena in Solid Nanostructures and Spintronics").

REFERENCES

1. S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).
2. B. L. Altshuler and A. G. Aronov, Solid State Commun. **46**, 429 (1983).
3. B. L. Altshuler and A. G. Aronov, in *Electron–Electron Interactions in Disordered Systems*, Ed. by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).
4. V. F. Gantmakher and V. T. Dolgopolov, Phys. Usp. **51**, 3 (2008).

5. A. K. Savchenko, A. S. Rylik, and V. N. Lutskii, Sov. Phys. JETP **58**, 1279 (1983).
6. G. M. Gusev, Z. D. Kvon, I. G. Neizvestnyi, and V. N. Ovsyuk, Sov. Phys. Semicond. **19**, 195 (1985).
7. S. V. Kravchenko, G. V. Kravchenko, J. E. Furneaux, et al., Phys. Rev. B **50**, 8039 (1994).
8. A. P. Mills, Jr., A. P. Ramirez, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **83**, 2805 (1999).
9. Y. Y. Proskuryakov, A. K. Savchenko, S. S. Safonov, et al., Phys. Rev. Lett. **89**, 076406 (2002).
10. A. Punnoose and A. M. Finkel'stein, Phys. Rev. Lett. **88**, 016802 (2002).
11. S. Anissimova, S. V. Kravchenko, A. Punnoose, et al., Nature Phys. **3**, 707 (2007).
12. Yu. G. Arapov, V. N. Neverov, G. I. Harus, et al., Semiconductors **41**, 1315 (2007).
13. G. M. Minkov, A. V. Germanenko, O. E. Rut, et al., Phys. Rev. B **75**, 193311 (2007).
14. S. V. Iordanskii, Yu. B. Lyanda-Geller, and G. E. Pikus, JETP Lett. **60**, 206 (1994).
15. M. I. D'yakonov and V. I. Perel', Sov. Phys. JETP **23**, 1053 (1971); Sov. Phys. Solid State **13**, 3023 (1971).
16. R. Winkler, Springer Tracts Mod. Phys. **191**, 1 (2003).
17. N. S. Averkiev, L. E. Golub, and G. E. Pikus, J. Exp. Theor. Phys. **86**, 780 (1998).
18. S. Pedersen, C. B. Sorensen, A. Kristensen, et al., Phys. Rev. B **60**, 4880 (1999).
19. S. I. Dorozhkin and E. B. Olshanetsky, JETP Lett. **46**, 502 (1987).
20. A. Zduniak, M. I. Dyakonov, and W. Knap, Phys. Rev. B **56**, 1996 (1997).
21. S. S. Murzin, S. I. Dorozhkin, G. Landwehr, and A. C. Gossard, JETP Lett. **67**, 113 (1998).
22. G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001).
23. G. M. Minkov, A. V. Germanenko, O. E. Rut, et al., Phys. Rev. B **85**, 125303 (2012).

Translated by R. Tyapaev