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On the Phase Boundaries of the Integer Quantum Hall Effect. Part II

S. S. Murzin

Institute of Solid State Physics, Russian Academy of Sciences, Chernogolovka, Moscow region, 142432 Russia
e-mail: murzin@issp.ac.ru

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It has been shown that the observation of the transitions between the dielectric phase and the integer-quantum-Hall-effect phases with the quantized Hall conductivity $\sigma_{xy}^{q} \geq 3e^2/h$ announced in a number of works is unjustified. In these works, the crossing points of the magnetic-field dependence of the diagonal resistivity ρ_{xx} at different temperatures T and $\omega_c\tau = 1$ have been misidentified as the critical points of the phase transitions. In fact, these crossing points are due to the sign change of the derivative $d\rho_{xx}/dT$ owing to the quantum corrections to the conductivity. Here, $\omega_c = eB/m$ is the cyclotron frequency, τ is the transport relaxation time, and m is the effective electron mass.

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The phase diagram of two-dimensional systems in the magnetic field has attracted the attention of both theorists and experimentalists for many years. Treating the integer quantum Hall effect (IQHE) in the context of the two-parameter scaling theory [1], which is graphically represented as a flow diagram [2, 3], yields the solution of the problem disregarding the electron–electron interaction. The further development of the scaling theory showed that the electron–electron interaction does not affect the position of the IQHE phase boundaries of a spin-polarized electron system [4, 5].

According to the scaling theory, the boundary between two IQHE phases is possible only if the quantized values of the Hall conductivity σ_{xy}^{q} of these two phases at zero temperature differ by e^2/h or (in the case of the spin degeneracy of the Landau levels) $2e^2/h$. However, a number of works reported on the observation of the transitions between the dielectric ($\sigma_{xy}^{q} = 0$) phase and the IQHE phases with the quantized Hall conductivity $\sigma_{xy}^{q} \geq 3e^2/h$. Song et al. [6] reported on the observation of the transition $\sigma_{xy}^{q} = 0 \longleftrightarrow \sigma_{xy}^{q} = 3e^2/h$ in two-dimensional hole systems in a strained Ge quantum well. The observation of the transitions $\sigma_{xy}^{q} = 0 \longleftrightarrow \sigma_{xy}^{q} \geq 3e^2/h$ in the two-dimensional hole systems in a strained Ge quantum well was also announced in [7]. Lee et al. [8] claimed the observation of the transitions $\sigma_{xy}^{q} = 0 \longleftrightarrow \sigma_{xy}^{q} = 6e^2/h$ and $\sigma_{xy}^{q} = 0 \longleftrightarrow \sigma_{xy}^{q} = 8e^2/h$ in doped AlGaAs/GaAs/AlGaAs quantum wells. Huang et al. [9] also reported on the observation of the transitions from the state with $\sigma_{xy}^{q} = 0$ to the states with $\sigma_{xy}^{q} = 6–16e^2/h$ in GaAs/AlGaAs heterojunctions. In all of these works [6–9], the crossing points of the magnetic-field dependence of the diagonal resistivity ρ_{xx} at $\omega_c\tau \approx 1$ and different temperatures T ($\omega_c = eB/m$ is the cyclotron frequency, τ is the transport relaxation time, and m is the effective electron mass) were considered as the critical points B_c of the phase transitions (see Fig. 1). In this case, ρ_{xx} weakly depends on the magnetic field and temperature near B_c.

In this work, it is shown that the above statements of the observation of the transitions between the dielectric phase and the IQHE phases with the quantized Hall conductivity $\sigma_{xy}^{q} \geq 3e^2/h$ are unjustified. In fact, the crossing point of the magnetic-field dependence of the diagonal resistivity $\rho_{xx}(T)$ at different temperatures is caused by the sign change of the derivative $d\rho_{xx}/dT$ owing to the quantum corrections [10] to the conductivity.

The classical diagonal and Hall conductivity in the magnetic field take the form

$$\sigma_{xx}^{0} = \frac{N_s e^2 \tau}{m} \frac{1}{1 + (\omega_c\tau)^2} \tag{1}$$

and

$$\sigma_{xy}^{0} = \frac{N_s e^2 \tau}{m} \frac{\omega_c\tau}{1 + (\omega_c\tau)^2}, \tag{2}$$

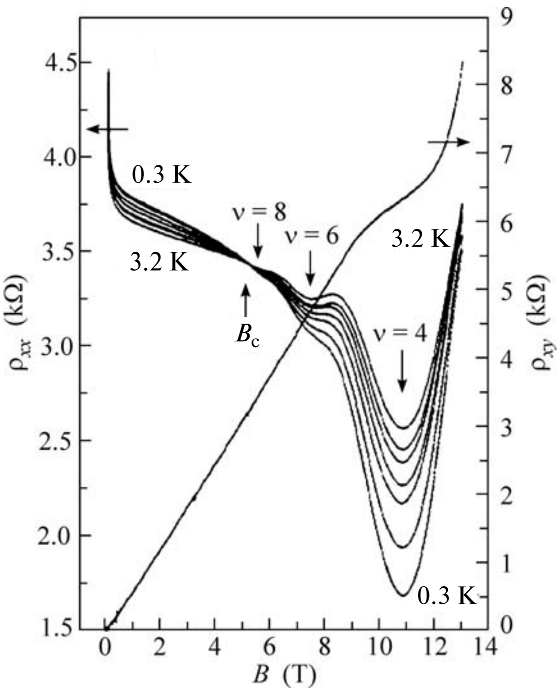

Fig. 1. Diagonal resistivity ρ_{xx} and the Hall resistivity ρ_{xy} of the doped AlGaAs/GaAs/AlGaAs quantum well. The electron density is $N_s = 1.04 \times 10^{16}$ m^{-2}. The temperatures for different ρ_{xx} values are 0.3, 0.5, 0.8, 1.2, 1.7, 2, and 3.2 K. The spin splitting is small and therefore invisible in ρ_{xx} and ρ_{xy}. The figure is taken from [8].

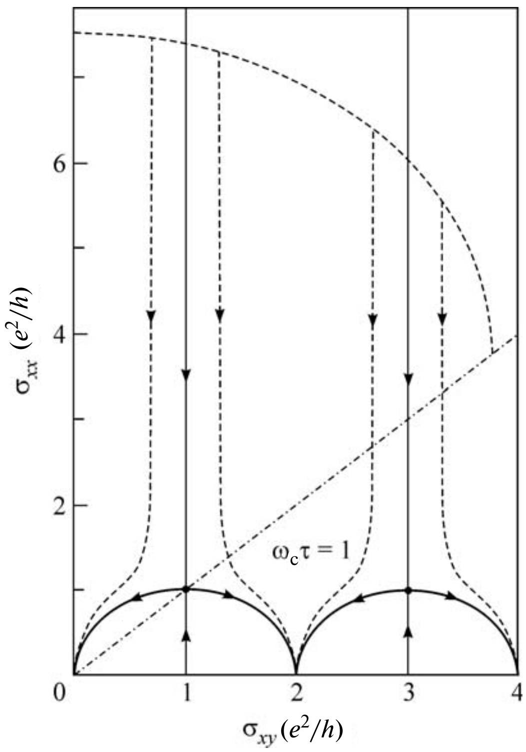

Fig. 2. Schematic scaling flow diagram for the quantum well with the parameters given in Fig. 1. The spin splitting is negligible. The solid lines are the separatrices of the diagram. The dashed lines are the bare conductivities σ^0_{xy} (σ^0_{xx}) at $\omega_c\tau < 1$ for the sample with the zero-field bare conductivity $\sigma^0 = 7.52e^2/h$. The dotted lines are the scaling flow lines. The dash–dotted straight line corresponds to $\omega_c\tau = 1$.

where N_s is the electron density. The quantum corrections $\Delta\sigma_{xx}(T) = \sigma_{xx}(T) - \sigma^0_{xx} \ll \sigma^0_{xx}$ to the diagonal conductivity decrease with temperature. At $T \longrightarrow 0$, $\sigma_{xx}(T) \longrightarrow 0$ except for the critical points, where $\sigma^0_{xy} = (i + 1/2)e^2/h$ (see Fig. 2). Excluding the weak-localization region ($B \lesssim 1$ T), the Hall conductivity σ_{xy} is independent of the temperature down to the temperatures at which $\sigma_{xx} \sim e^2/h$. At lower temperatures, the Hall conductivity depends on the temperature and approaches the nearest quantized integer value $\sigma_{xy}(B_i) = ie^2/h$ with $B_i < B_c$ at $\omega_c\tau < 1$.

Taking into account the quantum corrections, the diagonal and Hall resistivity of the two-dimensional electron system are given by the expressions

$$\rho_{xx}(T) = \rho^0_{xx} + [(\rho^0_{xy})^2 - (\rho^0_{xx})^2]\Delta\sigma_{xx}(T) \quad (3)$$

and

$$\rho_{xy}(T) = \rho^0_{xy} - 2\rho^0_{xx}\rho^0_{xy}\Delta\sigma_{xx}(T). \quad (4)$$

Here, we took into account that $\rho^0_{xx} = \sigma^0_{xx}/[(\sigma^0_{xx})^2 + (\sigma^0_{xy})^2]$ and $\rho^0_{xy} = \sigma^0_{xy}/[(\sigma^0_{xx})^2 + (\sigma^0_{xy})^2]$; σ^0_{xx}, σ^0_{xy}, ρ^0_{xx}, and ρ^0_{xy} are the bare (non-renormalized) values of the conductivity and resistivity, which correspond to the diffusion motion of electrons without the interference (localization) effects at distances longer than the diffusion step length. The temperature dependence of ρ_{xx} changes its sign in the magnetic field B such that $\rho^0_{xx}(B) = \rho^0_{xy}(B)$. In the classical treatment, $\rho^0_{xx}(B) = \rho^0_{xy}(B)$ at $\omega_c\tau = 1$.

At $\sigma_{xy} \gg e^2/h$ and $\omega_c\tau < 1$, the diagonal resistivity ρ_{xx} first increases with a decrease in the temperature, reaching the value

$$\rho_{xx,\,\max} = \frac{1}{2\sigma^0_{xy}}, \quad (5)$$

and then decreases and vanishes at $T \longrightarrow 0$ excluding the critical magnetic fields in which $\sigma^0_{xy} = (i + 1/2)e^2/h$. Thus, the negative value of the derivative $d\rho_{xx}/dT$ within the experimental range does not imply that the electron system is in a dielectric phase.

Note that the magnetic-field position of the IQHE phases at $\omega_c\tau \lesssim 1$ is not determined by the filling factor ν. Rather, it is given by the magnitude of $\sigma_{xy}^0 h/e^2$. This quantity is different from ν at $\omega_c\tau \lesssim 1$ [11].

Thus, we have shown that the statements [6–9] of the observation of the transitions between the dielectric phase and the IQHE phases with the quantized Hall conductivity $\sigma_{xy}^q \geq 3$ are unjustified. In fact, the crossing point of the magnetic-field dependence of the diagonal resistivity $\rho_{xx}(T)$ at different temperatures is caused by the sign change of the derivative $d\rho_{xx}/dT$ at $\omega_c\tau \approx 1$ owing to the quantum corrections to the conductivity.

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