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On the Phase Boundaries of the Integer Quantum Hall Effect

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It has been pointed out that, according to the two-parameter scaling theory, the magnetic-field position of the phases of the integer quantum Hall effect (IQHE) at $\omega_c\tau \lesssim 1$ is not determined by the filling factor $\nu = nh/eB$. The position of the IQHE phases is given by the bare Hall conductivity σ^0_{xy}. In this regard, it has been shown that the diagonal resistivity in the magnetic field measured by Sakr et al. [Phys. Rev. B **64**, 161308 (2001)] does not exhibit transitions between the $\sigma_{xy} = 3$, 4 and 6 IQHE states on the one hand and the dielectric state on the other hand in contrast to the assertion by Sakr et al.

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The phase diagram of the integer quantum Hall effect (IQHE) for many years has attracted the interest of both theorists and experimentalists. Interpretation of the IQHE on the basis of the two-parameter scaling theory [1], which is graphically represented as a flow diagram [2, 3], suggests the solution of the problem disregarding the electron–electron interaction. Further development of the scaling theory showed that the electron–electron interaction [4–7] does not affect the position of the IQHE phase boundaries.

According to the scaling approach, the boundary between two IQHE phases is possible only when the quantized values of the Hall conductivity σ^q_{xy} in these two phases at zero temperature differ by 1 or (in the case of spin degeneracy of the Landau levels) 2 in units of e^2/h. Sakr et al. [8] reported the observation of dielectric phases in p-SiGe heterostructures between the IQHE phases with the filling factors $\nu = 2$ and 3, 3 and 4, and 4 and 6. They supposed that $\nu = \sigma^q_{xy}$ and, consequently, the changes in σ^q_{xy} at the $3 \longleftrightarrow 0$, $4 \longleftrightarrow 0$, and $6 \longleftrightarrow 0$ phase transitions (0 denotes the dielectric state) are greater than 2, implying that the two-parameter scaling-theory interpretation of the IQHE is incorrect.

In this work, it is pointed out that, according to the two-parameter scaling theory, the magnetic-field position of the IQHE phases at $\omega_c\tau \lesssim 1$ is not determined by the filling factor ν. Here, $\omega_c = eB/m$ is the cyclotron frequency, τ is the transport relaxation time, and m is the electron effective mass. The positions of the phases are determined by the bare (see below) Hall conductivity σ^0_{xy}. In this regard, it is shown that the dependences of the diagonal resistivity ρ_{xx} on the magnetic field B presented in [8] do not exhibit the presence of the IQHE phases with $\sigma^q_{xy} = 3$, 4, and 6 and, the more so, the $3 \longleftrightarrow 0$, $4 \longleftrightarrow 0$, and $6 \longleftrightarrow 0$ transitions.

The bare (non-renormalized) Hall conductivity σ^0_{xy} corresponds to the diffusive motion of an electron without the interference (localization) effects at distances longer than the diffusion step length. It exhibits Shubnikov–de Haas oscillations with a change in the magnetic field [3]. According to the scaling theory, different IQHE phases of spinless electrons [1–3] are separated in the magnetic fields B_i, for which

$$\sigma^0_{xy}(B_i) = i + 1/2, \tag{1}$$

where $i = 0, 1, 2, 3, \ldots$. The Shubnikov–de Haas oscillations of σ^0_{xy} are small at $\omega_c\tau \lesssim 1$ and $\omega_c\tau \gg 1$, where $\sigma^0_{xy} \approx \nu$. For this reason, the crude consideration involves the classical expression for the bare Hall conductivity (in units of e^2/h)

$$\sigma^0_{xy} = \frac{n\tau h}{m}\frac{\omega_c\tau}{1+(\omega_c\tau)^2} = \nu\frac{(\omega_c\tau)^2}{1+(\omega_c\tau)^2}. \tag{2}$$

It follows from Eq. (2) that at $\omega_c\tau \lesssim 1$, $\sigma^0_{xy} \neq \nu$ and, consequently, the magnetic-field positions of the IQHE phase boundaries are different from the positions of the half-integer ν values at which the maxima of the Shubnikov–de Haas oscillations of the bare diagonal resistivity ρ^0_{xx} occur. The positions of the IQHE phase boundaries are found from the solution of Eqs. (1) and (2).

As an example, consider the two-dimensional system with the resistivity in zero magnetic field $\rho_0 =$

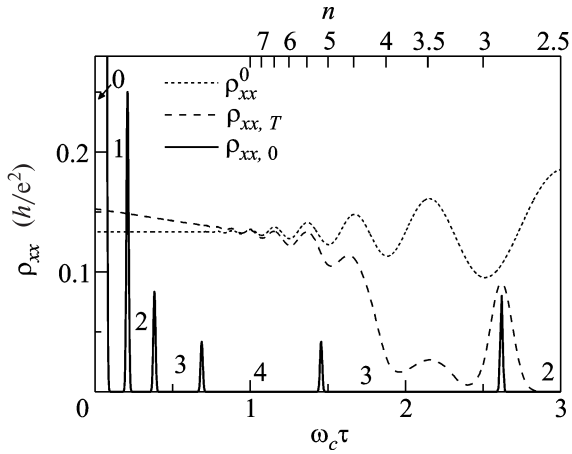
0.133 (in units of h/e^2), the Fermi energy $E_F = 7.5$ K, and the effective mass $m = 0.3m_0$ (m_0 is the free electron mass). In this case, the transport relaxation time is $\tau = \hbar/(\rho_0 E_F) = 7.64 \times 10^{-12}$ s and $n = mE_F/2\pi\hbar^2 = 4.05 \times 10^{14}$ m^{-2}. The figure shows the schematic drawings of (dotted line) the bare diagonal resistivity ρ^0_{xx}; (dashed line) the diagonal resistivity $\rho_{xx,T}$ with the inclusion of localization effects at finite temperature; and (solid line) the diagonal resistivity $\rho_{xx,0}$ at zero temperature as functions of $\omega_c\tau$. The upper scale in the figure is inversely proportional to ν. The lower scale $\omega_c\tau$ is constructed from the upper one with known n, τ, and m:

$$\omega_c\tau = \frac{nh\tau}{m}\frac{1}{\nu} = \frac{7.5}{\nu}. \tag{3}$$

The bare diagonal resistivity ρ^0_{xx} is plotted so that the Shubnikov–de Haas oscillation maxima and minima appear at half-integer and integer ν values, respectively. The oscillation amplitude is drawn arbitrarily. The diagonal resistivity at zero temperature $\rho_{xx,0}$ is zero everywhere except the IQHE phase boundaries. The positions of the peaks that appear at the phase boundaries were calculated according to Eqs. (1) and (2) and the amplitudes at the $\sigma^q_{xy} \longleftrightarrow \sigma^\sigma_{xy} + 1$ transitions [9] are $1/2\sigma^q_{xy}(\sigma^q_{xy} + 1)$. The positions of the peaks are different from the positions of the Shubnikov–de Haas oscillation maxima of the bare resistivity ρ^0_{xx}. In particular, the ρ^0_{xx} maxima at zero temperature and $\nu = 3.5$ and 4.5 appear in the IQHE region with $\sigma^q_{xy} = 3$ and $\rho_{xx,0} = 0$. The ρ^0_{xx} minimum at zero temperature and $\nu = 6$ lies in the minimum of the IQHE region with $\sigma^q_{xy} = 4$ and the IQHE region with $\sigma^q_{xy} = 6$ is absent.

To show how the curve $\rho^0_{xx}(\omega_c\tau)$ is transformed into $\rho_{xx,0}(\omega_c\tau)$ with an increase in the localization effects, the diagonal resistivity at finite temperature $\rho_{xx,T}$ is plotted. In strong magnetic fields, the curve $\rho_{xx,T}(\omega_c\tau)$ consists of the broadened peaks at the phase boundaries. In weak magnetic fields at $\omega_c\tau < 1$, $\rho_{xx,T} > \rho^0_{xx}$ due to quantum corrections. In the intermediate region ($1.5 < \omega_c\tau < 2.5$), the of minima and maxima Shubnikov–de Haas $\rho_{xx,T}$ are shifted downward with respect to the ρ^0_{xx} extrema.

Reasoning from the aforesaid, consider the results of Sakr et al. [8], who presented the magnetic-field dependence of the diagonal resistivity ρ_{xx} for p-SiGe heterostructures. They interpret the weak minima of ρ_{xx} as indications of the IQHE phases with the quantized values $\sigma^q_{xy} = \nu$ and claim the observation of



(Dotted line) The bare diagonal resistivity ρ^0_{xx}; (dashed line) the diagonal resistivity $\rho_{xx,T}$ with the inclusion of localization effects at finite temperature; and (solid line) the diagonal resistivity $\rho_{xx,0}$ at zero temperature schematically plotted as functions of $\omega_c\tau$. The upper scale in the figure is inversely proportional to ν. The numbers between the peaks of $\rho_{xx,0}$ are the values of the Hall conductivity at zero temperature σ^q_{xy} in different IQHE phases.

dielectric phases between the IQHE phases with $\nu = 2$ and 3, 3 and 4, and 4 and 6. At the same time, they assume that the IQHE phases are characterized by the filling factor and that σ^q_{xy} in these phases are equal to the nearest integer ν values. According to our consideration of the problem, this is not the case. At $\omega_c\tau \lesssim 1$, $\sigma^q_{xy} < \nu$ (see Eq. (2)). Although Sakr et al. [8] quoted only the $\rho_{xx}(B)$ curves, it may be shown that $\sigma_{xy} < 1.7$ always when $\nu > 1.2$, in particular, at $\nu = 3$, 4, and 6. Indeed,

$$\max[\sigma_{xy}] = \max\left[\frac{\rho_{xy}}{(\rho_{xx})^2 + (\rho_{xy})^2}\right]_{\rho_{xx} = \text{const}} = 1/2\rho_{xx}. \tag{4}$$

For $\nu > 1.2$, $\rho_{xx} > 0.29$ and thus $\sigma_{xy} < 1.7$. The value 1.7 is significantly smaller than 3, 4, and 6; therefore, there are no grounds to speak about the IQHE with $\sigma^q_{xy} = 3$, 4, and 6 and the transitions $3 \longleftrightarrow 0$, $4 \longleftrightarrow 0$, and $6 \longleftrightarrow 0$. According to the scaling theory, only the IQHE with $\sigma^q_{xy} = 1$ and 2 is possible in the samples under consideration.

To conclude, it has been showed that the magnetic-field position of the integer quantum-Hall-effect phases is determined by the bare Hall conductivity σ^0_{xy} rather than the filling factor. For $\omega_c\tau \lesssim 1$, $\sigma^0_{xy} \neq \nu$. The experimental results reported in [8] do not indicate the direct transitions from the dielectric phase to the integer quantum-Hall-effect phases with $\sigma^q_{xy} \geq 3e^2/h$.

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REFERENCES

1. H. Levine, S. B. Libby, and A. M.M. Pruisken, Phys. Rev. Lett. **51**, 1915 (1983).
2. D. E. Khmel'nitskii, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 454 (1983) [JETP Lett. **38**, 552 (1984)]; Phys. Lett. A **106**, 182 (1984); Helvetica Phys. Acta **65**, 164 (1992).
3. A. M. M. Pruisken, in *The Quantum Hall Effect*, Ed. by R. E. Prange and S. M. Girvin (Springer, Berlin, 1990).
4. A. M. M. Pruisken and I. S. Burmistrov, Ann. Phys. (N.Y.) **316**, 285 (2005).
5. A. M. M. Pruisken, M. A. Baranov, and I. S. Burmistrov, Pis'ma Zh. Eksp. Teor. Fiz. **82**, 166 (2005) [JETP Lett. **82**, 150 (2005)].
6. A. M. M. Pruisken and I. S. Burmistrov, Ann. Phys. (N.Y.) **322**, 1265 (2007).
7. A. M. M. Pruisken and I. S. Burmistrov, Pis'ma Zh. Eksp. Teor. Fiz. **87**, 252 (2008) [JETP Lett. **87**, 220 (2008)].
8. M. R. Sakr, Maryam Rahimi, S. V. Kravchenko, et al., Phys. Rev. B **64**, 161308 (2001).
9. S. S. Murzin, Pis'ma Zh. Eksp. Teor. Fiz. **88**, 374 (2008) [JETP Lett. **88**, 326 (2008)].

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