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Resistivity peak values at transition between fractional quantum Hall states

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Experimental data available in the literature for peak values of the diagonal resistivity in the transitions between fractional quantum Hall states ($\rho_{xx}^{\max}$) are compared with the theoretical predictions. It is found that the majority of the peak values are close to the theoretical values for two-dimensional systems with moderate mobilities.

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According to the microscopic [1, 2] and to the scaling [3] theories at transition between two quantum Hall (QH) states at low temperatures the diagonal (σ_{xx}) and Hall (σ_{xy}) conductivity components move on the (σ_{xy}, σ_{xx}) diagram along semicircles

$$\sigma_{xx}^2 + \left(\sigma_{xy} - \frac{p_1/q_1 + p_2/q_2}{2}\right)^2 = \left(\frac{p_1/q_1 - p_2/q_2}{2}\right)^2 \tag{1}$$

(σ_{xx} and σ_{xy} are in units of e^2/h). The semicircles connect pairs of points $(p_1/q_1, 0)$ and $(p_2/q_2, 0)$, corresponding to either an integer or fractional quantum Hall state, or an insulating state (p_1, p_2 are the integers and q_1, q_2 are the odd integers). Such semicircles has been observed experimentally [4] for the case of transition between quantum Hall state (1/3, 0) and insulating state (0, 0).

When the magnetic field changes the direct transition between two quantum Hall states with $\sigma_{xy} = p_1/q_2$ and $\sigma_{xy} = p_2/q_2$ is allowed at low temperature if and only if [5, 6]

$$|p_1q_2 - p_2q_1| = 1. \tag{2}$$

Therefore, σ_{xx} values in maxima of the transition peaks are equal to

$$\sigma_{xx}^{\max} = \frac{|p_1/q_1 - p_2/q_2|}{2} = \frac{1}{2q_1q_2}. \tag{3}$$

The semicircle (1) can be replotted on the (ρ_{xy}, ρ_{xx}) diagram by conventional transformation $\rho_{xy} = \sigma_{xy}/\left(\sigma_{xy}^2 + \sigma_{xx}^2\right)$ and $\rho_{xx} = \sigma_{xx}/\left(\sigma_{xy}^2 + \sigma_{xx}^2\right)$. Then all semicircles (1) but those starting from the point $(0, 0)$ transform into semicircles

$$\rho_{xx}^2 + \left(\rho_{xy} - \frac{q_1/p_1 + q_2/p_2}{2}\right)^2 = \left(\frac{q_1/p_1 - q_2/p_2}{2}\right)^2 \tag{4}$$

(ρ_{xx} and ρ_{xy} are in h/e^2 units). The semicircles (1) starting from the point $(0, 0)$ transform into vertical lines at $\rho_{xy} = 1, 3, 5 \ldots$. The values of ρ_{xx} at transition peak maxima are equal to

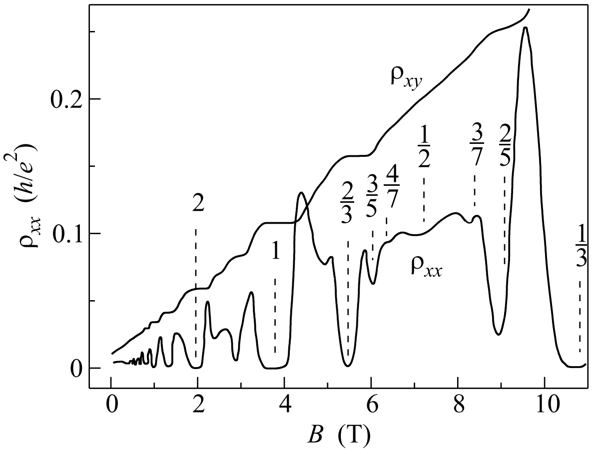
$$\rho_{xx}^{\max} = \frac{1}{2p_1p_2}. \tag{5}$$

In the present paper we verify relation (5) for appropriate experimental data available in the literature for different two-dimensional (2D) systems. The theories [1 – 3] have been developed for spinless (or totally spin polarized) electrons. Therefore, we discuss only transitions $p_1/q_1 \leftrightarrow p_2/q_2$, with p_1/q_1 and $p_2/q_2 \le 1$. For short $p_1/q_1 \leftrightarrow p_2/q_2$ denote the transition between QH states with $\sigma_{xy} = p_1/q_1$ and p_2/q_2. Note that spin depolarization of 2D electron system in GaAs/AlGaAs structures is possible even for $p_1/q_1 < 1$. Good agreement of the experimental data with the theoretical results is found only for samples with mobility $\mu \lesssim 2 \cdot 10^6$ cm^{-2}/Vs, and so, here we consider only such samples. The references are taken as the numbers of samples. The data for different samples or carrier densities from the same paper are marked additionally by the letters.

In Figure we show example (sample no.[7]^a) of the diagonal (ρ_{xx}) and Hall (ρ_{xy}) resistivities as a function of the magnetic field for the modulation-doped heterostructure GaAs/Al$_x$Ga$_{1-x}$As with electron density $n = 9 \cdot 10^{10}$ cm^{-2} and mobility (μ) about 10^6 cm^2/Vs. In the figure there is only one sufficiently clear defined peak at transition $2/5 \leftrightarrow 1/3$ with $1/2p_1p_2 = 0.25$ and the same experimental value of $\rho_{xx}^{\max} = 0.25$. Experimental values of $\rho_{xx}^{\max}$ which we have found in the literature are listed in Table. These data are chosen because they satisfy the following conditions: (i) the temperature is low, $T < 100$ mK; (ii) the transitions are pronounced (ρ_{xx} in both neighboring minima is smaller than $0.3\rho_{xx}^{\max}$), (iii) the transitions are sufficiently nar-

Values for the current carrier density (n or p), the mobility μ, and the height of the diagonal resistivity peak $\rho_{xx}^{\max}$. The references are taken as the numbers of samples. The data for the different samples or carrier densities from the same paper are marked additionally by the letters. Data [11]^{tilt} show the resistivity of sample [11] in tilted magnetic field

Transition →			1/3↔2/5	2/5↔3/7	1↔2/3	2/3↔3/5	3/5↔4/7
Value $1/2p_1p_2$ →			**0.25**	**0.0833**	**0.25**	**0.0833**	**0.0417**
Sample	Density (10^{10}cm^{-2})	Mobility (cm^{-2}/Vs)			$\rho_{xx}^{\max}$		
GaAs/AlGaAs [7]^a	$n = 9.0$	≈1· 10^6	0.25				
GaAs/AlGaAs [7]^b	$n = 15$	≈1·10^6				0.085	
GaAs/AlGaAs [7]^c	$n = 18$	1.9·10^6				0.074	
GaAs/AlGaAs [7]^d	$n = 19$	≈1·10^6			0.126		
GaAs/AlGaAs [8]	$n = 9.5$	1·10^6					0.042
GaAs/AlGaAs [9]	$n = 21$	2.2·10^5				0.068	
GaAs/AlGaAs [10]	$n = 6.6$	1.4·10^6		0.075			0.056
GaAs/AlGaAs [11]	$n = 2.4$	7·10^5	0.26			0.117	
GaAs/AlGaAs [11]^{tilt}	$n = 2.4$	7·10^5	0.25				
GaAs/AlGaAs [12]	$n = 35$					0.077	
GaAs/AlGaAs [13]	$n = 6.5$	5.5·10^5	0.33		0.25		
SiGe/Si/SiGe [14]	$n = 45$	7.5·10^4			0.22		
SiGe/Si/SiGe [15]	$n = 27$	2.5·10^5	0.22				
GaAs quantum well [16]	$p = 8.7$	$> 5 \cdot 10^5$			0.27		
GaAs quantum well [17]^a	$p = 1.2$				0.24		
GaAs quantum well [17]^b	$p = 1.5$				0.22		
GaAs/AlGaAs [18]	$p = 12$	5.4·10^5				0.077	
GaAs/AlGaAs [19]	$p = 16$	1.5·10^5	0.41		0.35	0.13	
GaAs quantum well [20]	$p = 15$	1·10^6		0.18	0.29	0.19	0.15



Magnetic field dependence of the diagonal (ρ_{xx}) and Hall (ρ_{xy}) resistivities for GaAs/Al$_x$Ga$_{1-x}$As heterostructure [7]^a with electron density $n = 9.0 \cdot 10^{10}$ cm^{-2} in a magnetic field perpendicular to the 2D electron system. Temperature $T = 30$ mK. Adopted from Ref.[7]

row (width of the peak at the level $2/3(\rho_{xx}^{\max})$ is smaller than $|B_{\max} - B_1|$ and $|B_{\max} - B_2|$, where $B_{\max}$ is the magnetic field in the maximum, B_1 and B_2 are the magnetic fields in the neighboring minima). Experimental data are disregarded if the current through the sample was rather large (> 20 nA), edge effects were knowingly essential [21, 22].

For GaAs/AlGaAs heterostructures the majority of the experimental values of $\rho_{xx}^{\max}$ are in agreement with the theoretical predictions $1/2p_1p_2$ with accuracy about 10%. Only the values of $\rho_{xx}^{\max}$ at transition $1 \leftrightarrow 2/3$ in sample [7]^d, at transition $2/3 \leftrightarrow 3/5$ in sample [11], at transition $3/5 \leftrightarrow 4/7$ in sample [10], and at transition $2/5 \leftrightarrow 1/3$ in sample [13] significantly differ from $1/2p_1p_2$. The differences at transitions $1 \leftrightarrow 2/3$ and $2/3 \leftrightarrow 3/5$ are due to spin depolarization. This is confirmed [11] by intense changing of the curve $\rho_{xx}(B)$ in tilted magnetic field [11]^{tilt} up to the field, where $p_1/q_1 = 4/7$. The difference $\rho_{xx}^{\max}$ from $1/2p_1p_2$ at transitions $3/5 \leftrightarrow 4/7$ in sample [10] is probably also defined by spin depolarization. Note, that at transition $3/5 \leftrightarrow 4/7$ in sample [8] $\rho_{xx}^{\max} = 0.042$ is nearly $1/2p_1p_2 = 0.0417$ and at transition $2/3 \leftrightarrow 3/5$ the values of $\rho_{xx}^{\max} \approx 1/2p_1p_2 = 0.0833$ in four samples [7]^b, [7]^c, [9], and [12]. Probably in these samples 2D electron systems are totally polarized in the regions of transition $3/5 \leftrightarrow 4/7$ and $2/3 \leftrightarrow 3/5$.

In two n-type $\mathrm{Si}_{1-x}\mathrm{Ge}_x/\mathrm{Si}/\mathrm{Si}_{1-x}\mathrm{Ge}$ quantum wells experimental values of $\rho_{xx}^{\max}$ are close to $1/2p_1p_2$ at transitions $1 \leftrightarrow 2/3$ observed in sample [14] and at transition $1/3 \leftrightarrow 2/5$ observed in sample [15].

In p-type GaAs quantum wells [16] and [17] with two different hole densities $[17]^a$ and $[17]^b$ $\rho_{xx}^{\max}$ values are close to the value of $1/2p_1p_2 = 0.25$ at transitions $1 \leftrightarrow 2/3$. Besides $\rho_{xx}^{\max} = 0.077 \approx 1/2p_1p_2 = 0.0833$ for 2D hole system [18] in $\mathrm{GaAs}/\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ heterostructure. For two p-type samples [19] and [20] $\rho_{xx}^{\max}$ values significantly differ from $1/(2p_1p_2)$, probably because the temperatures were not sufficiently low.

Disagreement between experimental and theoretical results for samples with mobility $\mu \gtrsim 2 \cdot 10^6$ cm^{-2}/Vs can be caused by insufficiently low temperature, by edge effects, or by weak macroscopic inhomogeneities of the sample [23].

In summary, at low temperature the peak values of the diagonal resistivity at the transitions between fractional quantum Hall states ($\rho_{xx}^{\max}$) are found to be close to the theoretically predicted values $1/2p_1p_2$ for majority of two-dimensional systems with moderate mobilities.

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