Hopping conductivity in heavily doped n-type GaAs layers in the quantum Hall effect regime

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We investigate the magnetoresistance of epitaxially grown, heavily doped n-type GaAs layers with thickness (40-50 nm) larger than the electronic mean free path (23 nm). The temperature dependence of the dissipative resistance R_{xx} in the quantum Hall effect regime can be well described by a hopping law ($R_{xx} \propto \exp\{-(T_0/T)^p\}$) with $p \approx 0.6$. We discuss this result in terms of variable range hopping in a Coulomb gap together with a dependence of the electron localization length on the energy in the gap. The value of the exponent $p \geqslant 0.5$ shows that electron-electron interactions have to be taken into account in order to explain the occurrence of the quantum Hall effect in these samples, which have a three-dimensional single electron density of states.

For a two-dimensional electron system it is well known that the discrete electron spectrum in a high magnetic field leads to quantized Hall resistance (Quantum Hall Effect). However, Landau quantization is not a strict prerequisite for the QHE. According to gauge arguments[1,2] it is sufficient that the dissipative conductance G_{xx} vanishes at the Fermi level, and that delocalized states exist below. The occurrence of the quantum Hall effect in not strictly two-dimensional systems has been considered by Khmelnitzkii[3] in conjunction with the scaling theoretical treatment of the QHE[4].

In our previous works[5,6], we observed the quantum Hall effect in a strongly disordered system, which consisted of a heavily Si-doped (n-type) GaAs layer between undoped GaAs. In this system a wide, smooth quantum well is formed by the impurity space charge potential that builds up at the layer interfaces. The electron gas is therefore confined inside the heavily doped GaAs layer, in the area of maximum disorder. The thickness d of the layers ranging from 50 up to 140 nm was larger than the electronic mean free path l of $15-30$ nm. The density of states (DOS) of noninteracting electrons in these samples is therefore expected to be practically three-dimensional. As the very strong disorder broadening in the samples leads to a rather smooth density of states without the formation of gaps between Landau levels even at the highest magnetic fields (≈ 20 T), we have proposed a reduction of the diagonal conductance G_{xx} due to electron-electron-interaction effects in diffusive transport as a possible explanation for the observed quantization of R_{xy} in the investigated, strongly disordered systems. Evidence for this explanation comes from the temperature dependence in the quantum Hall minima of G_{xx} in samples with a thickness between 50 and 140 nm[6], which is logarithmic with temperature T, and thus resembles the temperature dependence that is caused by quantum corrections due to electron-electron interactions in disordered conductors both in weak[7] and in high magnetic fields[8,9]. However, the logarithmic decrease of G_{xx} that was found in Ref.[6] exceeds in amplitude the range, where the theory of quantum correction[7–9] is applicable. Furthermore, in the thinnest sample (with a layer thickness $d = 50$ nm), which showed a fully developed quantization of the Hall effect, the conductance deviated from the logarithmic temperature dependence at the lowest temperatures for values of B where $G_{xx} \to 0$ and where the Hall conductance G_{xy} correspondingly shows a plateau at a value of $2e^2/h$.

In the current work we have investigated an additional number of strongly disordered GaAs layers with smaller values of d (namely 40 and 50 nm), showing a fully developed QHE below 100 mK. The obtained hopping law for the temperature dependent dissipative resistance R_{xx} is discussed in terms of the opening of a Coulomb gap.

The Hall conductance quantization in the aforementioned, quasi-three-dimensional systems with a "bare" (high temperature) conductance $G^0_{xx} \gg e^2/h$ can be understood qualitatively in the following way. Usually, in systems with coherent diffusive transport the dissipative conductance G_{xx} decreases with temperature T due to quantum corrections. The weak localization (single-particle) corrections are suppressed in a magnetic field B and reduce to[10]

$$G_{xx}(L_\varphi) = G^0_{xx} - \frac{2}{\pi^2}\frac{e^4}{h^2 G^0_{xx}}\ln(L_\varphi/L_0) =$$

$$G^0_{xx} - \frac{m}{\pi^2}\frac{e^4}{h^2 G^0_{xx}}\ln(T_1/T). \qquad (1)$$

Here $L_\varphi = \sqrt{D^0_{xx}\tau_\varphi}$ is the distance an electron moves diffusively during the phase breaking time $\tau_\varphi \propto T^{-m}$, D^0_{xx} is the "bare" high-temperature diffusion coefficient,

$$L_0 = d\sqrt{D^0_{xx}/D^0_{zz}} = \sqrt{dG^0_{xx}/\sigma_{zz}} \qquad (2)$$

is the electron displacement in the plane of the layer (perpendicular to the magnetic field) for the time of its diffusion across the layer (along the magnetic field), D_{zz} and σ_{zz} are the diffusion coefficient and the conductivity in the direction parallel to the magnetic field, T_1 is defined from the equation $d \approx \sqrt{2D^0_{zz}\tau_\varphi(T_1)}$. At low temperatures the phase breaks due to electron-electron interactions, leading to $m = 1$. The second order corrections in a magnetic field (Eq.1) are much smaller ($\pi h G^0_{xx}/e^2$ times) than the first-order corrections in zero field. Nevertheless, G_{xx} will eventually vanish, and in this case the Hall conductance G_{xy} should be quantized[1,2]. Since G_{xy} tends to different quantum values for different bare Hall conductances G^0_{xy}, transitional values of the bare conductance G^0_{xy} should exist, for which G_{xx} tends to a finite value and G_{xy} is not quantized.

This approach, initially developed for spinless noninteracting electrons, can give a reasonable, qualitative explanation for the occurrence of the quantum Hall effect with even numbers of quantization i in the above mentioned, strongly disordered GaAs layers[5,6]. Quantitative agreement with theory however does not exist because the quantum corrections (Eq.1) are small at real experimental conditions. To explain our results, we have proposed the inclusion of electron-electron interactions. In this case, the single-particle DOS and the conductance should decrease with decreasing temperature due to quantum corrections caused by interactions

$$G_{xx}(L_T) = G^0_{xx} - \frac{\lambda e^2}{\pi h}\ln(T_2/T) = G^0_{xx} - \frac{2\lambda e^2}{\pi h}\ln(L_T/L_0) \tag{3}$$

that occur both in weak[7] and in high magnetic fields[8,9]. Here $L_T \sim (D^0_{xx}\hbar/k_BT)^{1/2}$, k_B is the Boltzmann constant, and $T_2 \sim \hbar D^0_{zz}/k_Bd^2$. $\lambda \leqslant 1$ is the constant of interaction, which is of the order of unity and even somewhat larger in high magnetic fields ($\mu_B gB/k_BT \gg 1$) than in zero field (μ_B is the Bohr magneton). For $G^0_{xx} \gg e^2/h$ these corrections are much larger than the single-particle localization contributions (Eq.1). The interaction corrections (Eq.3) will lead to a vanishing of the dissipative conductance G_{xx} as a consequence of the opening of a Coulomb gap in the single particle DOS. Since also in this scenario G_{xx} will vanish at zero temperature, the Hall conductance should be quantized.

The samples used were prepared by molecular-beam epitaxy: on a GaAs (100) substrate the following layers were successively grown: an undoped GaAs layer (0.1 μm), a periodic structure of 30 × GaAs/AlGaAs(10/10 nm), an undoped GaAs layer (0.5 μm), the heavily Si-doped GaAs layer with a nominal thickness of $d = 40$ (sample 40) and 50 nm (sample 50) and donor(Si) concentrations of 1.5×10^{17} cm^{-3}, and last a cap layer of 0.5 μm GaAs (undoped). Samples with Hall bar geometries of a width of 0.2 mm and a length of 1.4 mm were etched out of the wafers. A phase sensitive ac-technique was used for the magnetotransport measurements down

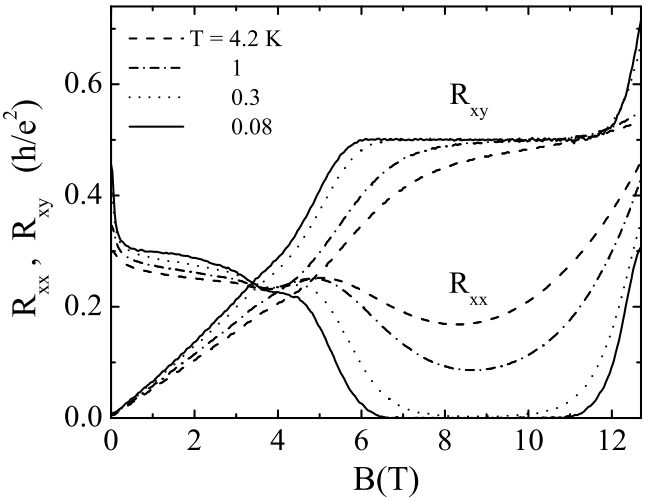

FIG. 1. Magnetic field dependence of the Hall (R_{xy}) and transverse (R_{xx}) resistance (per square) for sample 40 in a magnetic field perpendicular to the heavily doped GaAs layer at different temperatures

to 80 mK. In the experiments the applied magnetic field of up to 15 T was directed perpendicular to the layers. Samples from the same wafer showed identical behavior. The electron densities per square as derived from the slope of the Hall resistance R_{xy} in weak magnetic fields ($0.5 - 3$ T) at $T = 4.2$ K are $N_s = 4.5$ and 5.1×10^{11} cm^{-2}. The "bare" mobilities μ_0 are equal to 2500 and 2300 cm^2/Vs for sample 40 and 50 respectively, and the electron mean free path is about 23 nm for both samples. For the calculation of μ_0 we took the value of the bare resistance R_0 in the point of intersection of the curves $R_{xx}(B)$ for different temperatures at $B = 3.4$ T, taking into account that the classical resistance does not depend on field.

In Fig.1 the magnetotransport data, namely the Hall (R_{xy}) and transverse (R_{xx}, per square) resistance are plotted for sample 40 at temperatures below 4.2 K. The diagonal resistance R_{xx} decreases sharply at low magnetic fields due to the suppression of the weak localization corrections, and continues to decrease slightly between 0.5 and 4 T. It shows a deep minimum ranging from 6 to 11 T. The Hall resistance R_{xy} shows a linear increase up to 5 T, and then reveals a wide plateau from $B = 6$ T up to 11 T at the lowest temperatures with the value $R_{xy} = h/2e^2$ (i.e. $i = 2$), in the same field range where R_{xx} shows a deep minimum.

The Hall conductance $G_{xy} = R_{xy}/(R^2_{xx} + R^2_{xy})$ in the field range of $B = 0.5 - 4$ T does not depend on temperature. The diagonal conductance (per square) G_{xx} however shows a logarithmic temperature dependence with an only slightly field dependent coefficient, while the value of G_{xx} itself changes considerably. This behavior is in agreement with equation (3), giving an interaction constant $\lambda \approx 0.5$. The magnetotransport data for sample 50 are similar to the data for sample 40.

In our previous investigations of identical samples[5,6,12]

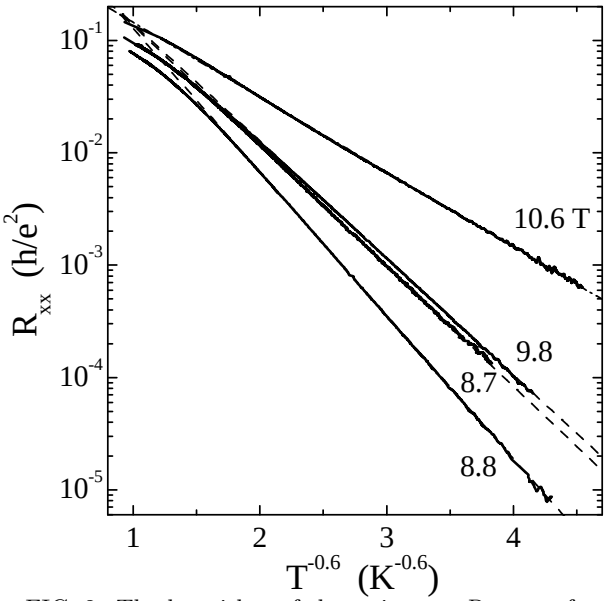

FIG. 2. The logarithm of the resistance R_{xx} as a function of $T^{-0.6}$ for sample 40 in the minimum ($B = 8.8$ T) and at larger fields indicated by lines, and for sample 50 in the minimum ($B = 8.7$ T)

with however a larger layer thickness, we found corrections to the conductivity due to electron-electron interactions. In a region of low magnetic field ($B < 4$ T) where $G^0_{xx} \gg e^2/h$ the magnetoresistance data can be quantitatively described in terms of quantum corrections due to electron-electron-interaction effects[12]. In high magnetic fields, even in samples with thicknesses d ranging up to 140 nm, quantization of the Hall conductance is observed. The mentioned samples show values of the bare conductance G^0_{xx} up to $2.6e^2/h$[5]. Even at these high fields the different QHE minima in the transverse conductance G_{xx} of different samples show a universal logarithmic temperature dependence in a large range of a rescaled temperature T/T_{sc}, where $T_{sc} \propto \exp(-3G^0_{xx}h/e^2)$[6]. Note however, that the decrease of G_{xx} is not small and that a logarithmic temperature dependence is observed beyond the region of applicability of the theory of quantum corrections[7]. In the thinnest sample ($d = 50$ nm) investigated in Ref.[6], showing a well pronounced QH plateau, a deviation from the logarithmic behavior becomes visible at the lowest temperatures ($T < 1$ K). It is this range of temperature and layer thickness, that the present work is focused on. We therefore study the temperature dependence of the resistance R_{xx} of samples with a thickness $d \leq 50$ nm, and therefore a rather low bare conductance G^0_{xx} of about e^2/h. These samples show a pronounced plateau in R_{xy} and a strong T-dependence near the minimum of R_{xx} at low temperatures, as shown in Fig. 1.

In Fig. 2 we plot the logarithm of the resistance R_{xx} as a function of $T^{-0.6}$ in the minima of R_{xx} corresponding to the plateaus at $R_{xy} = h/2e^2$ for samples 40 and 50[13], and additionally for sample 40 at somewhat larger B, but still not far from the minimum. The exponent $p = 0.6$ is chosen as a result of a fit of the experimental data to a hopping law

$$R_{xx} = R_0 \exp\{-(T_0/T)^p\} \tag{4}$$

in a range of temperature where $R_{xx}(T) < 0.1R_{xx}(4.2\text{ K}) \approx 0.02h/e^2$. The fitting parameters R_0 and T_0 are listed in the table.

Attempts to fit the data by an expression with a temperature dependent prefactor

$$R_{xx} = \alpha T^r \exp\{-(T_0/T)^p\} \tag{5}$$

and a fixed p different from 0.6 resulted in a less optimal fit. Moreover, the resulting fitting parameters are unphysical. For instance, for the case of $p = 0.5$ the fit gives $r = 0.65$, $\alpha = 17.1$ and $T_0 = 25.5$ K. For this situation, the prefactor αT^r in equation (5) at $T = 1$K corresponds to a conductance $G_{xx} = R_{xx}/R^2_{xy} = R_{xx}/(0.5h/e^2)^2 \approx 70e^2/h$ while $G_{xx} = 0.95e^2/h$ only at $T = 10$ K. The large prefactor in the conductance is compensated by a small exponential factor $\exp\{-(T_0/T)^p\} = \exp\{-25.5^{1/2}\} \approx 6.4 \times 10^{-3}$, while $G_{xx}(10\text{K})/G_{xx}(1\text{ K})$ has a value of about 3 only. The small difference between $G_{xx}(1\text{ K})$ and $G_{xx}(10\text{K})$ would be the result of a compensation of the two, which is not realistic. Thus we conclude, that the temperature dependence in the $i = 2$ minimum in R_{xx} is rather described by a hopping law according to Eq.(4) with a hopping exponent p near 0.6.

Without the existence of a Coulomb gap the Mott theory of variable range hopping[14] predicts the temperature dependence of R_{xx} to follow equation (4) with $p = 1/3$. According to the theory from Efros and Sklovskii[15,16], p is equal to 1/2 in the presence of a Coulomb gap around the Fermi energy E_F (both in zero magnetic field and in the QHE regime). This theory was developed for situations where the localization length ξ does not depend on the energy $\epsilon = |E - E_F|$ in the gap. In the case of Anderson localization the localization length ξ should depend on the energy ϵ near the Coulomb gap.

In the single-particle approach, at $G^0_{xy}(B) = ie^2/h$ with even i, the localization length ξ_{sp} of an electron at the Fermi level equals

$$\xi_{sp} \sim L_0 \exp\left(0.5\pi^2 {G^0_{xx}}^2 h^2/e^4\right) \tag{6}$$

estimated from the equation $G_{xx}(\xi_{sp}) = 0$ with G_{xx} taken from equation (1). According to the scaling theoretical treatment of the QHE, the localization length ξ_{sp} generally depends both on G^0_{xx} and G^0_{xy}. It diverges at $G^0_{xy}(B) = (i + 1/2)e^2/h$.

However, electron-electron interactions should result in a decrease of the localization length in the Coulomb gap. A lower limit of this decrease can be estimated from the equation $G_{xx}(\xi_0) = 0$ with G_{xx} taken from equation (3)

$$\xi_0 \sim L_0 \exp\left(\frac{\pi G^0_{xx}\, h}{2\lambda e^2}\right). \tag{7}$$

Outside the gap interaction is not important, and the localization length is equal or larger than the one given by expression (6) with $G^0_{xx} = G^0_{xx}(E)$ for the energy E. For typical values of $G^0_{xx} \approx e^2/h$ and $\lambda \approx 1$, ξ_0 is much smaller than ξ_{sp}. As it will be shown below, such an energy dependence of ξ should result in $p > 1/2$ in Eq.(4).

The single-particle density of states should be unaffected by an energy dependence of the localization length, unless the distance between electrons is much larger than the localization length, i.e. $g(\epsilon)\xi(\epsilon)^2|\epsilon| \ll 1$. It should still be linear: $g(\epsilon) = \gamma|\epsilon|$ with $\gamma = 2\kappa^2/\pi e^4$ (κ is the dielectric constant of the lattice). Let us suppose that in some range of energy $\xi = \alpha|\epsilon|^s$. Then by analogy with the Mott-law derivation[14,16] we obtain

$$R_{xx} \propto G_{xx} \propto \exp\{-(T_0/T)^{(s+1)/(s+2)}\}, \qquad (8)$$

where

$$T_0 = \frac{S}{k_B}\left(\frac{Ce^2}{\alpha\kappa}\right)^{1/(s+1)},$$

$$S = \left[(s+1)^{-(s+1)/(s+2)} + (s+1)^{1/(s+2)}\right]^{(s+2)/(s+1)}. \qquad (9)$$

The coefficient α depends on the magnetic field. For $s = 0$ equations (8) and (9) reduce to the results from Efros and Shklovskii. For $s \gg 1$ one finds activated behavior and $p = (s+1)/(s+2) = 0.6$ is obtained for $s = 1/2$. The main contribution to the conductivity is given by hopping electrons with an energy of

$$\epsilon_h = \left[\frac{Ck_Be^2(s+1)}{\alpha\kappa}T\right]^{1/(s+2)}$$

The corresponding localization length

$$\xi_h = \alpha|\epsilon_h|^s = \left[\frac{Ck_Be^2(s+1)}{\alpha\kappa}T\right]^{s/(s+2)}, \qquad (10)$$

of the electrons giving the main contribution to the conductivity for $T = 0.1$ K and $s = 1/2$ is listed in the table. The numerical coefficient C is taken to be 1.55 as defined from the equation for T_0 from the Efros-Shklovskii theory ($T_0 = 6.2e^2/\varepsilon\xi$)[17].

Since $\xi_h \propto T^{1/5}$ can not be smaller than ξ_0 it should become constant at the lowest temperatures and the temperature dependence should reduce to the Efros-Shklovskii law. In our experimental conditions ξ_h approaches $\xi_0 < 200$ nm at the very small temperature of $T < 3 \times 10^{-4}$ K at $B = 8.8$ T and $T < 5 \times 10^{-7}$ K at $B = 10.6$ T for sample 40.

A dependence of the localization length on energy could probably also account for hopping exponents $p > 1/2$, observed in zero-field experiments[18–20]. An energy-dependence as described above is also indicated by numerical simulations[21,22]. Therefore, also in zero magnetic field the power of T in equation (4) could be larger than $1/2$ in some range of temperature.

Sample	B (T)	T_0 (K)	R_0 (h/e^2)	ξ_h (μm)	ξ_T (μm)
40	8.8	6.0	2.41	0.63	0.24
40	9.8	4.3	1.55	0.95	0.4
40	10.6	2.04	0.66	2.3	1.2
50	8.7	4.5	1.6	0.9	0.37

TABLE I. Values of the magnetic fields B , the constant T_0 and the prefactor R_0, the localization length ξ_h of the electrons giving the main contribution to the conductivity, at $T = 0.1$ K and the localization length ξ_T of the electrons with energy $\epsilon/k_B = 0.1$ K.

In summary, in low magnetic fields (but still larger than 0.5 T) where $G_{xx} > 3e^2/h$, the temperature dependence of the diagonal conductance G_{xx} of heavily doped n-type GaAs layers with thicknesses ($d = 40 \div 140$ nm) larger than the mean free path of the electrons ($l = 23$ nm) is well described by the theory of quantum corrections due to electron-electron interactions. In high magnetic fields where $G_{xx} < 3e^2/h$ the temperature dependence of the conductance in the minima of $G_{xx,min}$ is still close to logarithmic down to $0.25e^2/h$, although the theory of quantum corrections is no more applicable. In the region of $G_{xx} < 0.25e^2/h$ the dissipative conductance shows an exponential decrease with a power $p \approx 0.6$, indicating the presence of a Coulomb gap. The data display the relevance of electron-electron interactions for the quantum Hall effect in these systems which have a 3D single-particle spectrum.

We have pointed out, that a dependence of the localization length on energy could result in an exponent $p > 1/2$ both in zero and nonzero magnetic field.

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