# Quantization of the Hall Conductance in a Three-Dimensional Layer

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Quantization of the Hall conductance is observed in epitaxial layers of heavily doped $n$-type GaAs with thickness ($\approx$100 nm) larger than both the screening length and the mean free path of the conduction electrons. Therefore, the single-particle spectrum of the electrons is three dimensional. Analogous to the quantum Hall effect in two-dimensional systems, the magnetic field $B$ dependence of the Hall conductance $G_{xy}$ shows, below 1 K, steps at quantized values $ie^2/h$ for $i = 2, 4, 6$ together with pronounced minima in the transverse conductance $G_{xx}$. The minima in $G_{xx}$ and $|dG_{xy}/dB|$ appear in magnetic fields where, at $T = 4$ K, $G_{xy} = ie^2/h$. [S0031-9007(98)05631-2]

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For the occurrence of the quantum Hall effect (QHE) in a two-dimensional (2D) electron system in a magnetic field, the dissipative components of the conductivity tensor have to be zero with the existence of delocalized electron states below the Fermi level [1]. In a single-particle description, the Landau quantization of the electrons leads to the Landau-level structure with localized states in the gaps between the levels resulting in the integer QHE. The gaps in the electronic density of states can also be caused by electron-electron correlations leading to the fractional QHE in systems with weak disorder [1]. The QHE was also observed in strongly anisotropic systems like a superlattice [2] or an organic metal [3–5] which have quasi-2D character due to the only weak coupling between 2D conducting layers. In these cases there are also gaps in the density of states in high magnetic fields and the Hall conductance $G_{xy}$ is quantized to $G_{xy} = 2iNe^2/h$ as in $N$ independent 2D parallel layers ($i$ is a small integer and $N \gg 1$). In the superlattice a gap occurs when the cyclotron energy $\hbar\omega_c = \hbar eB/m$ in a magnetic field $B$ becomes larger than the interlayer tunneling integral $t$. In the organic metal a spin density wave state and, therefore, a gap appear due to electron-electron interaction.

In this work we observed, for the first time, the quantization of the Hall conductance in epitaxial layers of heavily doped $n$-type GaAs with thickness (100 nm) larger than the screening length (10 nm) and the mean free path $l$ (15–30 nm) of the electrons determined from mobility. Ignoring electron-electron correlation effects, the electronic system in the investigated GaAs layer has a 3D energy spectrum like that for bulk material (in particular there are no variations of the density of states due to dimensional quantization across the layer). However, at low temperatures the system becomes 2D for coherent phenomena in the diffusive transport. For the explanation of the observed phenomenon, we will discuss the possibility that the corrections in the single-particle density of states at the Fermi level and in the diagonal components of the conductivity tensor due to electron-electron interactions in disordered systems [6] become of the order of unity in high magnetic fields [7,8]. Therefore, one can expect that a Coulomb gap occurs in the density of states and that the diagonal components of the conductivity tensor vanish.

The samples used were prepared by molecular-beam epitaxy. On a GaAs (100) substrate the following were successively grown: an undoped at low temperature grown GaAs layer (0.1 $\mu$m), an undoped GaAs layer (0.6 $\mu$m), a periodic structure 20 $\times$ GaAs/AlAs (10/5 monolayers), an undoped GaAs layer (1 $\mu$m), the heavily Si-doped GaAs of nominal thickness $d = 100$ nm and different donor (Si) concentrations (1, 1.5, and 3 $\times 10^{17}$ cm$^{-3}$ for three samples), and a cap layer (0.1 $\mu$m for sample 1 and 1 $\mu$m for the other samples). Hall bar geometries of width 0.18 mm and length 3 mm were etched out of the wafer. A phase sensitive ac technique was used for the magnetotransport measurements. In most experiments the applied magnetic field up to 23 T was directed perpendicular to the layers. A few experiments were done at 4.2 K with the field oriented in the plane of the layers but still perpendicular to the applied current.

The bulk density of electrons $n$ in a heavily doped layer has been determined from the periodicity of the Shubnikov–de Haas oscillations in the transverse resistance $R_{xx}$ with the results as listed in Table I for the three investigated samples. The mobilities $\mu$ have been determined from the zero-magnetic-field resistance and the Hall resistance $R_{xy}$ in the linear region of weak magnetic fields at $T = 4.2$ K (see Table I). They are close to the

TABLE I. Values for the electron concentration $n$, the mobility $\mu$, and the thickness $d$ of the investigated heavily doped GaAs layers.

| Sample | $n$ (cm$^{-3}$) | $\mu$ (cm$^2$/V s) | $d$ (cm) |
|---|---|---|---|
| 1 | $0.82 \times 10^{17}$ | 1900 | $0.7 \times 10^{-5}$ |
| 2 | $1.8 \times 10^{17}$ | 2400 | $0.7 \times 10^{-5}$ |
| 3 | $2.5 \times 10^{17}$ | 2500 | $0.83 \times 10^{-5}$ |

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mobilities found for bulk material with comparable electron densities [9]. The average thickness $d$ of the conducting layers (see Table I) has been obtained by dividing the electron density $N_s$ per area unit determined from the Hall measurements by the volume density $n$. The values for $d$ of the investigated samples are somewhat smaller than the nominal 100-nm thickness.

In order to be sure about the three dimensionality of the electronic spectrum of the investigated layers, we have measured the magnetoresistance of samples 1 and 2 at $T = 4.2$ K in a parallel magnetic field up to 11.5 T additionally to the measurements in perpendicular field. The curves of the transverse resistance for the two field configurations (see Fig. 1 for sample 2) show the usual Shubnikov–de Haas oscillations. The stronger increase at the extreme quantum limit (EQL) where only the lowest Landau level is occupied was also observed for the two configurations (for sample 1 above 5 T and for sample 2 above 9 T). These curves are close to each other, pointing to the 3D character of the spectra. The smaller oscillation amplitude for the magnetic field parallel to the layer can be ascribed to the finite ratio of the sample thickness $d$ to the magnetic length. Pronounced oscillations can be observed only when the electron orbit radius is much smaller than $d$. For one sample prepared with a metallic gate on top, we could reduce the thickness of the conducting region. Although $R_{xx}$ and $R_{xy}$ almost doubled due to the reduced thickness of the heavily doped GaAs layer, the positions of maxima and minima of the Shubnikov–de Haas oscillations did not change indicating a constant 3D density of electrons. These measurements confirm that the electron spectra in the samples are really three dimensional at $T = 4.2$ K.

In Fig. 1 the magnetotransport data of the Hall resistance $R_{xy}$ and the transverse resistance $R_{xx}^{\square}$ have been plotted for sample 2 at a few temperatures between 4.2 K and 60 mK. The transverse resistance per square $R_{xx}^{\square}$ has been obtained from the measured resistance $R_{xx}$ taking account for the surface geometry of the Hall bar. The magnetoresistance data at 4.2 K show the typical behavior of bulk material with weak Shubnikov–de Haas oscillations for increasing field and a stronger upturn in the EQL where only the lowest Landau level is occupied. At lower temperatures the Hall resistance reveals remarkable steps near the values $h/e^2 i$ with $i = 2$ and 4. In the corresponding fields pronounced minima can be observed in the transverse resistance. Similar structures could be observed for the other two samples investigated. For sample 1 one step for $i = 2$ could be observed at $H = 7.2$ T. For sample 3, only weak oscillations around values $i = 4, 6$, and 8 at 11, 14, and 20 T, respectively. Comparing the observed plateaus in $R_{xy}$ and minima in $R_{xx}^{\square}$ with those in the QHE for a two-dimensional sample, one notes that neither the steps are completely flat nor the minima are very deep. Similar structures would be observed in the QHE for a not yet fully developed gap in the density of states.

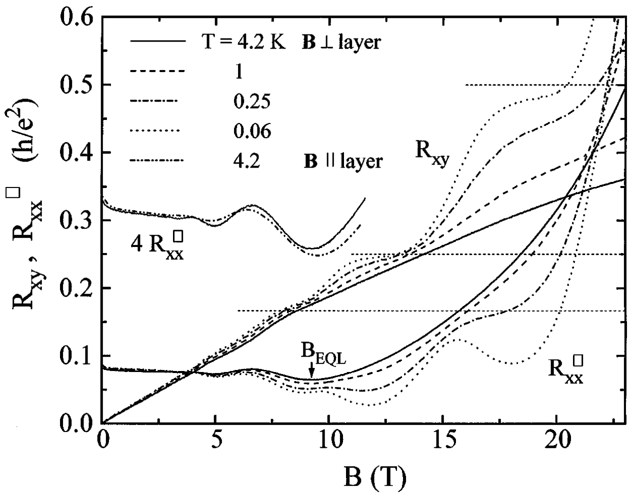


FIG. 1. Magnetic field dependence of the Hall resistance $R_{xy}$ and the transverse resistance $R_{xx}^{\square}$ (per square) for sample 2 in a magnetic field perpendicular to the GaAs layer at different temperatures. The dashed horizontal lines represent quantized values $R_{xy} = h/e^2 i$ for $i = 2$, 4, and 6. The arrow indicates the field $B_{\rm EQL}$ of the extreme quantum limit. The reproduced data of $4R_{xx}^{\square}$ at 4.2 K show a comparison with $B$ parallel to the layer.

In Fig. 2, we have plotted the Hall conductance $G_{xy}$ and the transverse conductance $G_{xx}^{\square}$ per square for sample 2 as obtained by inverting the magnetoresistance tensor in the usual way, i.e., $G_{xy} = R_{xy}/(R_{xx}^{\square 2} + R_{xy}^2)$ and $G_{xx}^{\square} = R_{xx}^{\square}/(R_{xx}^{\square 2} + R_{xy}^2)$. The Hall conductance $G_{xy}$ practically does not depend on temperature in magnetic fields up to $B_{\rm EQL}$, the field where only the lowest Landau level is occupied. In higher fields the Hall conductance starts to oscillate around the curve for $T = 4.2$ K and all low-temperature curves cross this curve at the quantized values $ie^2/h$. At the same field values minima in

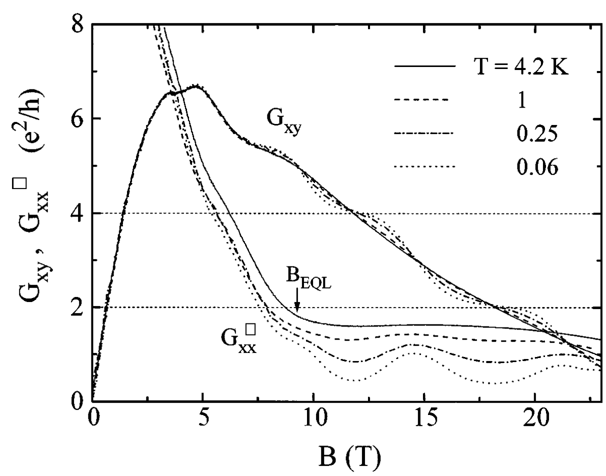


FIG. 2. Magnetic field dependence of the Hall conductance $G_{xy}$ and the transverse conductance $G_{xx}^{\square}$ (per square) for sample 2 at different temperatures. The horizontal dashed lines represent quantized values $G_{xy} = ie^2/h$ for $i = 2$ and 4.

$G_{xx}^{\square}$ arise which become more pronounced at decreasing temperature. Similar phenomena have been observed for the other two samples.

In order to illustrate the quantized behavior in the Hall conductance for all our samples we have plotted in Fig. 3 $G_{xx}^{\square}$ as a function of $G_{xy}$ for magnetic field sweeps at 4.2 K and 60 mK. For low magnetic fields (6–9 T, for sample 2) the curves descend more or less vertically due to a strong dependence of $G_{xx}^{\square}$ on magnetic field, while $G_{xy}$ changes slowly. For increasing fields, the 60-mK curves show for decreasing conductance $G_{xx}^{\square}$ a quantized behavior at $G_{xy} = ie^2/h$ for $i = 2$, 4, 6, and 8. By comparing these graphs for the different samples, one can observe that for samples 1 and 2 with equal conductance $G_{xx}^{\square}$ at 4.2 K the same minimum value occurs in $G_{xx}^{\square}$ at 60 mK. Sample 3, showing a higher 4.2-K conductance, has less pronounced minima at $i = 4$ and 6 compared to samples 1 and 2. The same property can be observed by comparing the data at $i = 4$, 6, and 8 of sample 3. From these findings we can conclude that the lower the value of $G_{xx}^{\square}$ at 4.2 K, the more pronounced the minimum in $G_{xx}^{\square}$ at 60 mK at the quantized Hall conductance values. We note that, for a thicker sample with $d = 2.7 \times 10^{-5}$ cm and with comparable electron density and mobility but larger $G_{xx}^{\square}$ (around $6e^2/h$ in the EQL) compared to the data in Fig. 3, we did not observe the quantized behavior for temperatures down to 50 mK.

In Fig. 4 we have plotted the temperature dependence of $G_{xx}^{\square}$ for sample 1 at $B = 7.2$ T in the minimum of $G_{xx}^{\square}$ (corresponding to $i = 2$ quantization in $G_{xy}$) and for sample 2 at 11.7 T ($i = 4$) and at 18.1 T ($i = 2$). All three dependencies show a very similar behavior, both with respect to the range of values and the functional dependence. The dependence is practically linear in $T^{2/3}$ scale between 0.1 and 0.25 K and, moreover, the $T^{2/3}$ dependence extrapolates to zero conductance for zero temperature. Because of the limited temperature range another power law dependence $T^{1/2}$ could equally well describe our experimental results. However, the zero-temperature extrapolation for a $T^{1/2}$ power law extrapolated to a negative $G_{xx}^{\square}$ for zero temperature. In order to check for a possible activated behavior, we also plotted $\ln G_{xx}^{\square}$ versus $1/T$. These curves are not linear in the temperature range 0.1–0.25 K. Below 0.1 K we observed in our $G_{xx}^{\square}$ data a saturating behavior. A reduction of the excitation current did not influence this saturating behavior.

In a two-dimensional electron system the conductivity minimum occurs at integer values for the filling factor $\nu = N_s h/eB$ corresponding to the number of occupied Landau levels below the Fermi level for an electron density per unit area $N_s$. For the investigated GaAs layers the values of the filling factor are appreciably larger than $i$ using the electron density $N_s$ as determined from the Hall measurements at low fields. For example, for sample 2 with $N_s = 1.3 \times 10^{12}$ cm$^{-2}$, $\nu = 4.6$ and 3 in fields of 11.7 and 18.1 T, respectively, while $i = 4$ and 2.

The experimental results cannot be explained by the manifestation of oscillations in the density of states due to dimensional quantization across a layer (i.e., a division of the lowest Landau subband into levels of dimensional quantization) because of the smallness of the mean free path of the electrons with respect to the layer thickness. For the occurrence of dimensional quantization the electrons should move ballistically across the layer. In our case the mean free path in zero magnetic field (15–30 nm) is a few times smaller than the thickness of the samples. In the extreme quantum limit of the applied magnetic fields the mean free path along the field is even less than the

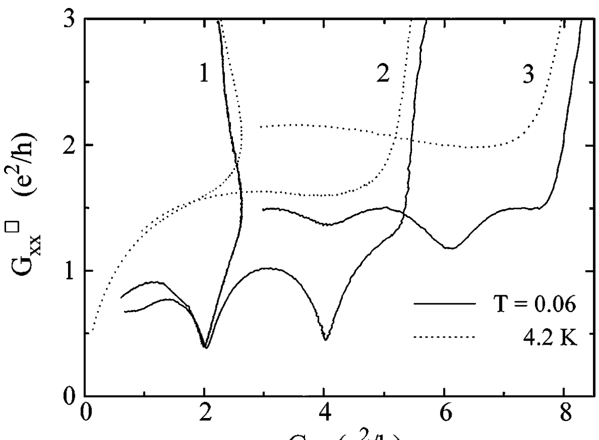


FIG. 3. Transverse conductance $G_{xx}^{\square}$ as a function of the Hall conductance $G_{xy}$ for the three samples at 0.06 and 4.2 K as constructed from the dependence of $G_{xx}^{\square}$ and $G_{xy}$ on magnetic field.

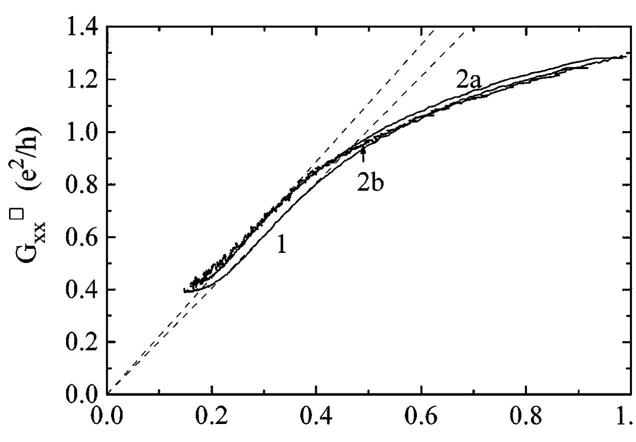


FIG. 4. Temperature dependence of the transverse conductance $G_{xx}^{\square}$ at constant magnetic field in the minima of $G_{xx}^{\square}$, for sample 1 at 7.2 T (where $G_{xy}$ has a plateau at $2e^2/h$) and for sample 2 at 11.7 and 18.1 T (plateaus in $G_{xy}$ at $4e^2/h$ and $2e^2/h$).

mean free path in zero magnetic field [10]. Because of the disorder the broadening of dimensionally quantized levels is larger than the energy difference between the levels. In the case of a pure layer with $d = 0.7 \times 10^{-5}$ cm and $N_s = 1.3 \times 10^{12}$ cm$^{-2}$ as in sample 2, three levels (each of them with two spin orientations) of dimensional quantization would be occupied in a 11.7-T field (at the minimum of $G_{xx}^{\square}$) with the energy difference between the third and second level of the dimensional quantization, i.e., $(3^2 - 2^2)(\pi\hbar/d)^2/2m$, corresponding to 68 K. From the transport electron relaxation time $\tau$ the broadening $\Gamma = \hbar/\tau$ corresponds to $\approx$90 K. The calculated electron spectrum for a simple model of broadened 2D levels transforms already for a broadening $\Gamma$ larger than 50 K into a 3D spectrum at the Fermi level. Moreover, the monotonic magnetic-field dependence of the resistance above $B_{\mathrm{EQL}}$ at 4.2 K indicates that the electron spectrum is really three dimensional at this temperature (which is much smaller than the corresponding energy difference between the levels of dimensional quantization) without variations of the density of states due to dimensional quantization.

In view of the necessity of zero-valued dissipative components of the conductivity tensor for the occurrence of the Hall conductance quantization, we will discuss the following possibility. In 3D disordered metallic systems the electron-electron interaction yields small corrections to the single-particle density of states and to the conductivity [6]. Applied magnetic fields increase these corrections to the single-particle density of states and to the diagonal elements of the conductivity tensor due to the decrease of the diffusion coefficients across the field at $\omega_c\tau \gg 1$ without any effect on the Hall conductivity [6]. This was confirmed experimentally in transport data on tellurium-doped bismuth [11]. Extrapolation of the results to the EQL [7,8] reveals that these corrections can become of the order of unity. For the case of the investigated layers, the effect of the electron-electron interaction should be even larger due to limited diffusion of an electron across the layer and, therefore, one may expect that a Coulomb gap occurs and diagonal components of the conductivity tensor disappear in high magnetic fields, while the Hall conductivity is finite. The importance of the electron-electron interaction for our samples is revealed by the fact that in low magnetic fields $G_{xx}^{\square}$ slightly decreases with temperature while $G_{xy}$ is temperature independent (see Fig. 2).

We note that the electron-electron correlation problem becomes two dimensional for $T < T_0 = l v_F \hbar/k_B d^2$, when the correlation length $L_T = (D_{\parallel}\hbar/k_B T)^{1/2}$ ($D_{\parallel} = v_F l$ is the diffusion coefficient along magnetic field, and $v_F$ the electron velocity at the Fermi level) becomes larger than the thickness of the layer $d$. Below this temperature ($T_0 \approx 3$ K for sample 2 in a 11.7-T field), we observed a power-law temperature dependence of $G_{xx}^{\square}$ in the minima which could follow from a Coulomb gap with zero density of states only at the Fermi level. In the case of a real gap the temperature dependence should be exponential as in the integer and fractional QHE in weakly disordered 2D systems.

In accordance with the gauge argument by Laughlin [12] a disappearance of dissipative components of the conductivity tensor should result in the Hall conductance quantization. However, there is a question why the dissipative conductivity and the single-particle density of states at the Fermi level vanish only at particular fields. This has to result from a possible quantization of the many-body electron system, which would be different from the usual Landau quantization. To some extent the situation is similar to the situation in the fractional QHE where gaps at the Fermi level occur due to the quantization of the many-body electron system in high magnetic fields.

In summary, we observed the Hall conductance quantization in three-dimensional GaAs layers below 1 K in the extreme quantum limit of the applied magnetic field, while the usual Shubnikov–de Haas oscillations do not depend on temperature below 4 K. We suggest that the quantization is related to the enhancement of electron-electron interaction effects in a disordered electron system in the lowest Landau level.

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[1] *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girven (Springer-Verlag, Berlin, 1990).
[2] H. L. Störmer, J. P. Eisenstein, A. C. Gossard, W. Wiegmann, and K. Baldwin, Phys. Rev. Lett. **56**, 85 (1986).
[3] J. R. Cooper *et al.*, Phys. Rev. Lett. **63**, 1984 (1989).
[4] S. T. Hannahs *et al.*, Phys. Rev. Lett. **63**, 1988 (1989).
[5] L. Balicas *et al.*, Phys. Rev. Lett. **75**, 2000 (1995).
[6] B. L. Al'tshuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys. JETP **50**, 968 (1979)].
[7] S. S. Murzin, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 45 (1986) [JETP Lett. **44**, 56 (1986)].
[8] S. S. Murzin and A. G. M. Jansen, J. Phys. Condens. Matter **4**, 2201 (1992).
[9] Kh. I. Amirkhanov, P. I. Bashirov, and A. Yu. Mollaev, Fiz. Tekh. Poluprovodn. **4**, 1884 (1970) [Sov. Phys. Semicond. **4**, 1616 (1970)].
[10] P. N. Argyres and E. N. Adams, Phys. Rev. **104**, 900 (1956).
[11] S. S. Murzin, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 45 (1985) [JETP Lett. **44**, 56 (1985)].
[12] R. B. Laughlin, Phys. Rev. B **23**, 5632 (1981).