Effect of hole–hole scattering on the conductivity of the two-component 2D hole gas in GaAs/(AlGa)As heterostructures

S. S. Murzin and S. I. Dorozhkin
Institute of Solid State Physics, 142432 Chernogolovka, Moscow Region, Russia

G. Landwehr
Physikalisches Institut der Universität Würzburg, D-97074 Am Hubland, Würzburg, Germany

A. C. Gossard
University of California, Santa Barbara, CA 93106, USA

(Submitted 23 August 1997)
Pis'ma Zh. Éksp. Teor. Fiz. **67**, No. 2, 101–106 (25 January 1998)

The temperature dependences of the zero-magnetic-field resistivity ρ and magnetoresistance of the 2D hole gas in GaAs/(AlGa)As heterostructures are investigated in the temperature interval 0.4–4.2 K. As the temperature T is increased, (i) the resistivity ρ grows with a decreasing derivative $d\rho/dT$, and (ii) the positive magnetoresistance diminishes from about 40% at $T=0.4$ K to about 1% at $T=4.2$ K. The results are explained in terms of a temperature-dependent mutual scattering of the holes, accompanied by momentum transfer between two different spin-split subbands.
[S0021-3640(98)00302-8]

PACS numbers: 73.40.Kp, 73.50.Jt

A positive magnetoresistance of up to 40% in weak magnetic fields has been observed at low temperatures in the high-mobility 2D hole gas of GaAs/(AlGa)As heterostructures in studies going back many years.[1–7] At first this magnetoresistance was attributed to two-carrier conduction.[3] It is known there are two groups of holes with different spectra and mobilities in the 2D hole systems of GaAs/(AlGa)As heterostructures. These two groups are formed from the heavy hole band as a result of the lifting of the spin degeneracy by the spin–orbit interaction in the absence of inversion symmetry.[8–10] In such systems a positive magnetoresistance should be observed[11] both in the case of the elastic scattering of holes by impurities and in the inelastic scattering of holes by phonons (even in the presence of inter-group scattering[12]).

However, in the experiments of Refs. 2, 4, and 7 the magnetoresistance was found to be strongly temperature dependent even at relatively low temperatures, when the electron–phonon scattering is unimportant. The magnetoresistance decreases with temperature, almost vanishing[2,7] at $T=4.2$ K. This has raised doubts that the effect is due to two-carrier conduction,[4,6] and new ideas have been put forward. In Refs. 2 and 4 it was noted that a qualitatively similar effect can be caused by weak localization in a system with strong spin–orbit coupling.[13] However, the weak localization effects are too small to account for the large magnetoresistance in highly conductive heterostructures.[2] The authors of Ref. 6 hypothesized that the magnetoresistance could originate from quantum corrections due to the hole–hole interaction in disordered systems at values of the inverse screening length q_s that are large compared to the hole wave numbers at the Fermi level k_F. It can be shown that the magnetoresistance in this case is also small. The authors of Ref. 7 assumed that the magnetoresistance should be suppressed if the thermal energy k_BT (k_B is Boltzmann's constant) is much larger than the energy separation between the two bands at the wave vector of the smaller Fermi circle Δ_F. However, our calculations show that this factor alone cannot suppress the magnetoresistance but only leads to some changes in its value. Moreover, a drastic decrease in the magnetoresistance is observed when $k_BT \ll \Delta_F$. This factor alone obviously contradicts the idea of the authors of Ref. 7. Thus there is no satisfactory explanation of the strong temperature dependence of the magnetoresistance of the high-mobility 2D hole gas in GaAs/(AlGa)As heterostructures.

In this paper we propose a new idea which is capable of explaining this phenomenon: the mutual scattering of holes belonging to different groups. The equations derived here are compared with both the results of our detailed study of the temperature dependence of the zero-magnetic-field resistance and magnetoresistance and with all the available data; the results of this comparison demonstrates that the proposed effect gives a reasonable explanation of the data. It is important to note that the temperature dependence of the mutual scattering was found to be proportional to T^2, which supports the basic idea.

1. EFFECT OF HOLE–HOLE SCATTERING

The positive magnetoresistance in a system with two groups of carriers is caused by the difference between their drift velocities $\mathbf{u}_i$ in an electric field. Intense mutual scattering of carriers should equalize the velocities leading to a vanishing magnetoresistance. Equations introducing the mutual scattering into the transport problem have been derived previously[15–18] for the case when the inter-group scattering is absent. Here we use these equations to calculate the zero-magnetic-field resistance and magnetoresistance for the case of carriers with like charges and different mobilities. Although they should not be expected to describe the magnetoresistance very accurately, we hope that they will describe rather well the main features of the phenomenon. The equation of motion in an electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ for particles of group 1, taking into account the collisions with particles of group 2, has the form[16,17]

$$m_1\mathbf{u}_1/\tau_1+\eta n_2(\mathbf{u}_1-\mathbf{u}_2)=e\mathbf{E}+(e/c)(\mathbf{u}_1\times\mathbf{H}). \tag{1}$$

A similar equation can be written for the particles of group 2. Here m_i are the effective masses, τ_i are the momentum relaxation times for each group, and η is the mutual friction coefficient

$$\eta=\frac{m_1m_2}{m_1n_1+m_2n_2}\,\frac{1}{\tau_{e-e}}. \tag{2}$$

Since the relaxation time τ_{e-e} of the relative drift velocity $\mathbf{u}_1-\mathbf{u}_2$ due to the mutual scattering of carriers is proportional to T^{-2} (Ref. 21; see also the Appendix), η can be written as

$$\eta=\alpha T^2. \tag{3}$$

By solving the system of equations for $\mathbf{u}_i$ and substituting these velocities into the expression for the current density $\mathbf{j}=n_1e\mathbf{u}_1+n_2e\mathbf{u}_2$, we find the conductivities σ_{xx} and σ_{xy}:

$$\sigma_{xx}=\frac{[nw(He/c)^2+(\eta nw+w_1w_2)(\eta n^2+n_1w_2+n_2w_1)]e^2}{(He/c)^4+[n^2\eta^2+2\eta(n_1w_2+n_2w_1)+w_1^2+w_2^2](He/c)^2+(\eta nw+w_1w_2)^2}, \tag{4}$$

$$\sigma_{xy}=\frac{n(He/c)^2+(\eta^2n^3+2\eta n(n_1w_2+n_2w_1)+(n_1w_2^2+n_2w_1^2))}{(He/c)^4+[n^2\eta^2+2\eta(n_1w_2+n_2w_1)+w_1^2+w_2^2](He/c)^2+(\eta nw+w_1w_2)^2}\frac{e^3}{c}H. \tag{5}$$

Here $w_i=m_i/\tau_i=e/\mu_i$, $n=n_1+n_2$, and $w=(w_1n_1+w_2n_2)/n$. The longitudinal and Hall resistivities are $\rho_{xx}=\sigma_{xx}/(\sigma_{xx}^2+\sigma_{xy}^2)$ and $\rho_{xy}=\sigma_{xy}/(\sigma_{xx}^2+\sigma_{xy}^2)$. At low temperatures, when $\tau_{e-e}\gg\tau_i$ $(n\eta\ll w)$, the conductivity is the sum of the conductivities of each group. In this case our equation for the magnetoresistance coincides with the equation given in Ref. 11. The magnetoresistance is positive and saturates in high magnetic fields $\mu_iH/c\gg1$. At high temperatures, when $\tau_{e-e}\ll\tau_i$, the longitudinal resistivity ρ_{xx} and Hall resistivity ρ_{xy} are equal to $\rho_{xx}=1/ne\mu$, $\rho_{xy}=H/nec$. Here $\mu=e/w$ is the average mobility. In this case, the magnetoresistance is absent and the zero-magnetic-field resistivity ρ does not change with temperature if the μ_i are temperature independent. In the intermediate range $\tau_{e-e}\sim\tau_i$ the temperature dependence of the resistance exists only in weak magnetic fields $\mu_iH/c\leqslant1$. The resistivity ρ_{xx} increases with temperature and saturates at high temperatures. The difference $\rho(T\to\infty)-\rho(T=0)$ is equal to the difference $\rho_{xx}(H\to\infty,T=0)-\rho(T=0)$.

2. EXPERIMENT

The two samples used in the experiment were prepared by molecular-beam epitaxy. Sample 1 consisted of a GaAs (100) substrate overgrown with the following layers: undoped GaAs (0.2 μm), a GaAs(20Å)/$Al_{0.26}Ga_{0.74}As$(20Å) periodic structure (20 periods), undoped GaAs (1 μm), undoped $Al_{0.26}Ga_{0.74}As$ (250 Å), $Al_{0.26}Ga_{0.74}As$ doped with Be to $\sim2.7\times10^{18}$ cm^{-3} (300 Å), and undoped GaAs (50 Å). Sample 2 differed from sample 1 by the content of Al in $Al_xGa_{1-x}As$ layers ($x=0.3$), by the thickness of the doped AlGaAs layer, which was equal to 200 Å, and by the presence of a cap layer which consisted of 150 Åof undoped $Al_{0.3}Ga_{0.7}As$ and 100 Å of undoped GaAs.

The densities n_1 and n_2 for the two different groups of holes were determined from the Shubnikov–de Haas oscillations at a low temperature and are listed in Table I. This procedure is similar to the one used in Refs. 1 and 3. In fields $H<1$ T the period of the oscillations is determined by the density n_1 of the holes with the lower mass and density. Above 2 T the period is determined by the total hole density n. For samples 1 and 2 the total densities are 3.23×10^{11} and 3.43×10^{11} cm^{-2}, and the average mobilities μ at $T=4.2$ K are 7.4×10^4 and 9.3×10^4 cm^2/V·s, respectively.

TABLE I.

Sample	n_1, cm^{-2}	n_2, cm^{-2}	$\mu_{1,0}$, $cm^2/V\cdot s$	$\mu_{2,0}$, $cm^2/V\cdot s$	α, $g\cdot cm^2/s\cdot K^2$	β, 1/K
1	1.14×10^{11}	2.09×10^{11}	22×10^4	5.4×10^4	3.7×10^{-29}	0
2	1.27×10^{11}	2.16×10^{11}	24.7×10^4	7.5×10^4	2.85×10^{-29}	—
2	1.27×10^{11}	2.16×10^{11}	24.6×10^4	7.7×10^4	2.54×10^{-29}	0.02

The temperature dependence of the resistivity at $H=0$ is shown in Fig. 1. Both samples show qualitatively similar behavior. The resistivity increases with temperature by about 40%, with a derivative $d\rho/dT$ that is largest at low temperatures. Up to $T\approx3$ K the derivative $d\rho/dT$ decreases and then starts to increase slightly. The magnetoresistance at different temperatures is shown in Figs. 2 and 3. The main effect, common to both samples, is a positive temperature-dependent magnetoresistance with a tendency to saturation at high magnetic fields. The magnetoresistance strongly decreases as the temperature increases from 0.4 to 4.2 K.

3. DISCUSSION

The hole–hole scattering explains both the strong decrease of magnetoresistance at high temperatures and the temperature dependence of the zero-magnetic-field resistivity, with decreasing $d\rho/dT$ observed at $T<3$ K. The quantum corrections due to weak localization[13] and the hole–hole interaction[14] in our samples should be smaller than 1%. The large value of $\Delta/k_B\approx10$ K (Refs. 8 and 10) contradicts the explanation given in Ref. 7.

We fitted the experimental data by Eqs.(3)–(5) by varying three unknown parameters, namely, the temperature-independent mobilities $\mu_{1,0}=e/w_1$, $\mu_{2,0}=e/w_2$ and α, trying to reach the best accuracy at low temperatures. The results of the fitting are shown in Figs. 1, 2, and 3. The chosen values of parameters are listed in Table I. The main features of the experimental data are described well by the fitting curves.

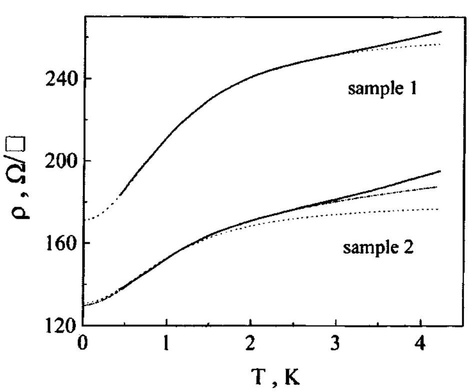

FIG. 1. Resistivity at zero magnetic field versus temperature. The solid lines are experimental curves, the dotted and dot-dashed lines show theoretical fits with temperature-independent and temperature-dependent hole mobilities, respectively.

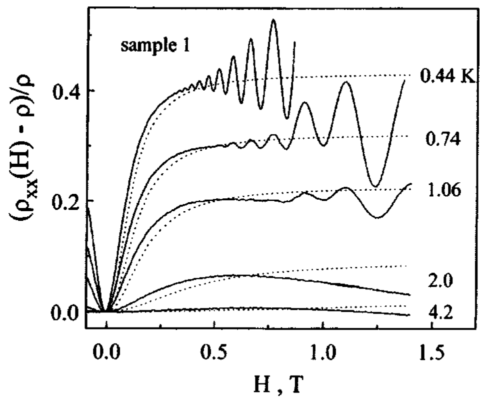

FIG. 2. Magnetoresistance $(\rho_{xx}-\rho)/\rho$ of sample 1 at different temperatures in a magnetic field perpendicular to the plane of the sample. The solid lines are experimental curves, the dotted lines represent the results of fitting.

There are several effects which were not taken into account by our simple model. These are inter-group scattering (for the case of elastic scattering this effect was considered in Ref. 12) and anisotropy of the hole Fermi surface. While the former effect can be suppressed in the case of elastic scattering by remote impurities (separated from the two-dimensional system by a spacer), it definitely exists in the case of the hole–hole scattering. These effects may be responsible for some discrepancies between the experimental and the theoretical magnetoresistance curves. The temperature dependence of the zero-magnetic-field resistivity should be much less sensitive to these factors. The differences between the fitting and the experimental curves observed in Fig. 1 at high temperatures can be explained by the temperature dependence of the mobilities μ_i due to electron–phonon scattering and to the finite value of k_BT/E_F (E_F is the Fermi energy, $E_F/k_B \approx 20$ K). The biggest correction caused by the latter effect is linear in k_BT/E_F because of the temperature dependence of the screening:[22]

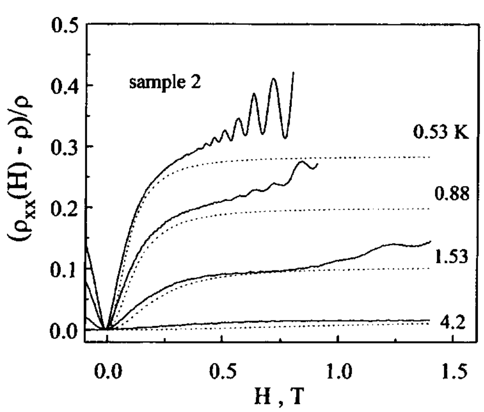

FIG. 3. Magnetoresistance $(\rho_{xx}-\rho)/\rho$ of sample 2 at different temperatures. The solid lines are experimental curves, the dotted lines represent the results of fitting.

$$\mu_i^{-1} = \mu_{i,0}^{-1}(1 + \beta_i T). \tag{6}$$

This effect is important only for scattering with momentum transfer close to $2\hbar k_{F,i}$ ($k_{F,i}$ are the hole wave numbers at the Fermi level) and, therefore, is strongly dependent on the presence of the corresponding harmonics in a particular scattering potential. It can be very different for different samples even with similar structures. The fitting of the data with temperature-dependent mobilities given by Eq. (6), where we take $\beta = \beta_1 = \beta_2$, yields considerably better results for sample 2 (see Fig. 1). The results for sample 1 were not changed (for this sample β was found to be close to zero). New fitting parameters for sample 2 are also listed in Table I. The calculated magnetoresistance curves changed only slightly after taking into account the corrections to μ_i and we therefore do not present the new curves. The coefficients β have reasonable values smaller than $k_B / E_F \approx 0.05\ \mathrm{K}^{-1}$. It is worth noting that at $T = 4.2$ K the differences between the experimental curves and the new fitting curves in Fig. 1, which we ascribe to the electron–phonon scattering, are approximately equal for the two samples.

In order to verify whether η is proportional to T^2 we tried to fit the temperature dependence of the resistivity taking $\eta = \alpha T^p$ with $p = 1.5$ and 2.5 in the temperature range 0.4–3 K. In both cases the agreement with the experiment was noticeably worse in comparison with the case $p = 2$.

There are neither experimental nor theoretical data on η, α or τ_{e-e} in a two-component 2D electron (hole) gas. In Ref. 21, where the dependence $\tau_{e-e} \propto T^2$ was derived, the factor mutliplying T^2 was not calculated. In order to understand whether the values of α obtained from the fitting are reasonable or not, we have calculated τ_{e-e} and η for a simple model, following the approach of Ref. 18. This model neglects the anisotropy of the real energy spectrum and assumes the absence of hole transitions from one subband to the other. Although these conditions are not fulfilled in our system, we believe that the calculated value has the correct order of magnitude. Under the condition $q_s = e^2(m_1 + m_2)/\kappa_0 \hbar^2 \gg k_{iF}$ ($q_s / \max(k_{iF}) \approx 10$ in GaAs/(AlGa)As heterostructures with $n = 3 \times 10^{11}\ \mathrm{cm}^{-2}$) we have

$$\eta = \frac{8}{3\hbar^3} \left(\frac{m_1 m_2}{m_1 + m_2} \right)^2 \frac{1}{n_1 n_2} \ln \frac{\sqrt{n_1} + \sqrt{n_2}}{\sqrt{n_1} - \sqrt{n_2}} (k_B T)^2. \tag{7}$$

For the case of the effective masses $m_1 = 0.2 m_e$ and $m_2 = 0.8 m_e$ calculated in Refs. 8 and 10 we have obtained $\alpha \approx 7 \times 10^{-29}\ \mathrm{g \cdot cm^2/s \cdot K^2}$ for our samples, which is in reasonable agreement with the experimental values.

We have checked that the published results on the temperature-dependent magnetoresistance for p channels in GaAs/AlGaAs heterostructures are consistent with our explanation. Unfortunately, a detailed comparison is not possible because, to the best of our knowledge, the only experimental data for which the temperature range was large enough to demonstrate strong variation of the magnetoresistance is given by Fig. 4 of Ref. 7. But in this paper only the total hole density $n = 2.08 \times 10^{11}\ \mathrm{cm}^{-2}$ is presented. Nevertheless, we can approximately determine a coefficient $\alpha \approx 1 \times 10^{-28}\ \mathrm{g \cdot cm^2/s \cdot K^2}$ for these data because it is not very sensitive to the n_1 / n_2 ratio. The data presented in Fig. 5 of the same paper[7] for a low-mobility sample in the temperature range 0.3–1.3 K show only a weak temperature dependence of the magnetoresistance, which implies that τ_{e-e} is much less than the elastic scattering time and gives no chance of determining α.

Estimation of α is possible for the data presented in Fig. 2 of Ref. 4 ($n=3.8\times10^{11}$ cm^{-2}, $n_1=1.01\times10^{11}$ cm^{-2}), although the variation of the magnetoresistance there is not large there. This estimation gives $\alpha\approx1\times10^{-29}$ g$\cdot$cm^2/s$\cdot$K^2. The variation of α with hole density is consistent with the expected dependence (see Eq. (A8)) at least qualitatively.

In conclusion, we have shown that the temperature dependence of both the zero-magnetic-field resistance and the magnetoresistance of the 2D hole gas in GaAs/(AlGa)As heterostructures is governed by the hole–hole scattering at low temperatures. Similar effects can exist in other high-mobility semiconductor systems which contain several groups of carriers with different mobilities.

We acknowledge a helpful discussion with V. T. Dolgopolov and D. V. Shovkun. This work has been supported by Russian State Program ''Nanostructures'' (Grant 1-085/4) and by Deutsche Forschungsgemeinshaft.

[1] H. L. Stormer, Z. Schlesinger, A. Chang *et al.*, Phys. Rev. Lett. **51**, 126 (1983).
[2] E. E. Mendez, W. I. Wang, L. L. Chang, and L. Esaki, Phys. Rev. B **30**, 1087 (1984).
[3] J. P. Eisenstein, H. L. Stormer, V. Narayanamurti *et al.*, Phys. Rev. Lett. **53**, 2579 (1984).
[4] E. E. Mendez, Surf. Sci. **170**, 561 (1986).
[5] Y. Iye, E. E. Mendez, W. I. Wang, and L. Esaki, Phys. Rev. B **33**, 5854 (1986).
[6] Zhou Haiping, Zheng Houzhi, Yang Fuhua *et al.*, Chin. Phys. **10**, 223 (1990).
[7] M. Henini, P. A. Crump, P. J. Rodgers *et al.*, J. Cryst. Growth **150**, 446 (1995).
[8] U. Ekenberg and M. Altarelli, Phys. Rev. B **30**, 3369 (1984).
[9] D. A. Broido and L. J. Sham, Phys. Rev. B **31**, 888 (1985).
[10] T. Ando, J. Phys. Soc. Jpn. **54**, 1528 (1985).
[11] J. M. Ziman, *Principles of the Theory of Solids*, Cambridge University Press, 1972.
[12] E. Zaremba, Phys. Rev. B **42**, 14143 (1992).
[13] S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).
[14] P. A. Lee and T. V. Ramakrishman, Rev. Mod. Phys. **57**, 287 (1985).
[15] C. A. Kukkonen and P. F. Maldaldague, Phys. Rev. Lett. **37**, 782 (1976).
[16] V. F. Gantmakher and Y. B. Levinson, Zh. Éksp. Teor. Fiz. **74**, 261 (1978) [Sov. Phys. JETP **47**, 133 (1978)].
[17] V. F. Gantmakher and Y. B. Levinson, *Carrier Scattering in Metals and Semiconductors*, Vol. 19 of Modern Problems in Condensed Matter Sciences, edited by V. M. Agranovich and A. A. Maradudin, Elsevier–North-Holland, 1987.
[18] J. Appel and A. W. Overhauser, Phys. Rev. B **18**, 758 (1978).
[19] A. N. Friedman, Phys. Rev. **159**, 553 (1967).
[20] A. H. Thompson, Phys. Rev. Lett. **35**, 1786 (1975).
[21] C. S. Ting, A. K. Ganguly, and W. Y. Lai, Phys. Rev. B **24**, 3371 (1981).
[22] A. Gold, V. T. Dolgopolov, J. Phys. C **18**, L463 (1985); Phys. Rev. B **33**, 1076 (1986).
[23] F. Stern, Phys. Rev. Lett. **18**, 546 (1967).

Published in English in the original Russian journal. Edited by Steve Torstveit.