

---

# Experimental Implementations of the Superconductor-Insulator Transition

V.F. Gantmakher

Institute of Solid State Physics RAS, Chernogolovka, Russia [gantm@issp.ac.ru](mailto:gantm@issp.ac.ru)

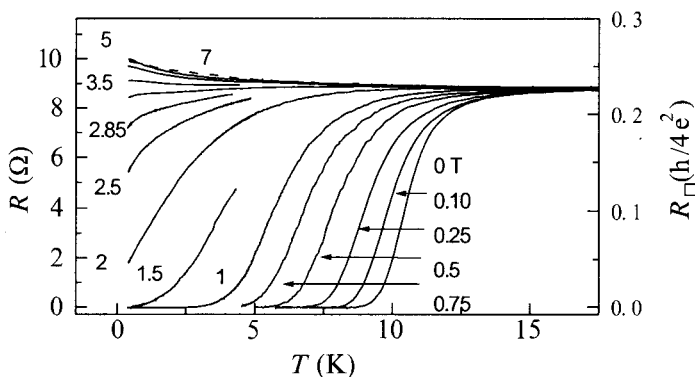
**Summary.** Various experimental observations of the superconductor-insulator transition are described and compared with two theoretical models: one based on boson-vortex duality and the other where the superconducting fluctuations at low temperatures in the magnetic field are calculated. The latter shows that the superconducting fluctuations in dirty but homogeneous superconductor act as grains in a granular superconductor.

When superconductivity is destroyed by changing some of parameters, either intrinsic (carriers density, level of disorder) or extrinsic (magnetic field) ones, the material can turn not only into normal metal but into insulator as well. We'll discuss here magnetic-field-induced superconductor-insulator transitions (SIT). Magnetic field transfers the superconductor into insulator in the case when the carrier density in the material is low and the level of disorder is high, so that without the superconductivity the material would be in zero field on the insulating side of the metal-insulating transition. The main sign of SIT is the fan-like set of the resistance curves  $R(T)$ : they go down with decreasing of the temperature at fields below the critical,  $B < B_c$ , and go up at fields  $B > B_c$ .

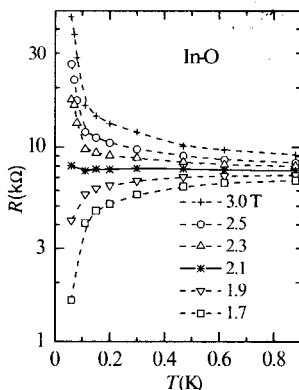
The list of materials which displayed such type of behaviour contains amorphous  $\text{Mo}_x\text{Ge}_{1-x}$  [1] and  $\text{Mo}_x\text{Si}_{1-x}$  [2] films, amorphous  $\text{InO}_x$  films [3, 4], ultrathin films of Be [5], crystalline films of  $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_{4+y}$  [6, 7]. Two typical examples of such sets of curves relevant to different limits are presented in Figs. 1 and 2. In  $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_{4+y}$  (Fig. 1) the growth of the resistance with decreasing temperature on the non-superconducting side of the field-induced transition was below ten percent so that it reminded more a metal with quantum corrections to its conductivity than an insulator. In amorphous  $\text{InO}_x$  (Fig. 2), typical for insulator exponential temperature dependence of the resistance resulted in almost tenfold increase of the resistance.

All the above-listed experiments were interpreted as SIT in two-dimensional (2D) electron systems. This interpretation for those systems where the growth of the resistance was comparatively trifling was based on scaling hypothesis [8] which asserts that there is no non-superconducting metallic state at zero temperature in 2D. Hence any state of a film which does not display tendency to become superconducting, i.e. which has negative derivative  $\partial R/\partial T < 0$ , should be accepted as insulating.

The theoretical grounds for existence of 2D-SIT which was suggested in [9, 10] appealed to the boson-vortex duality model. It considered the superconducting phase as a condensate of Cooper pairs with localized vortices and the insulating phase as a condensate of vortices with localized Cooper pairs. The theory described only vicinity of the SIT and predicted existence of some critical region on the  $(T, B)$ -plane where the behavior of the system was



**Fig. 1.** Temperature dependence of the resistivity of  $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_{4+y}$  films at different magnetic fields [7]. At high fields the resistivity changes lie in the range of 10%; two upper curves cross at low temperatures.



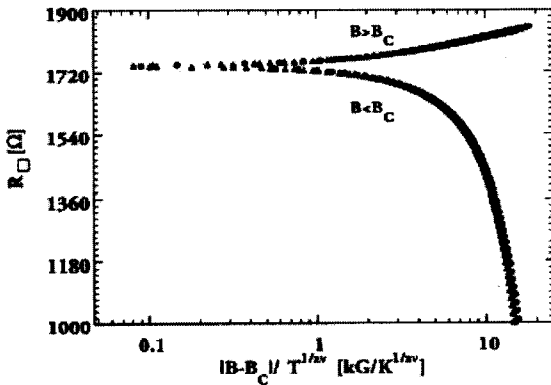
**Fig. 2.** Temperature dependence of the resistivity of amorphous  $\text{InO}_x$  films at different magnetic fields [4].

governed by competition of the quantum phase transition correlation length  $\xi \propto (B - B_c)^{-\nu}$  and thermal length  $L_T \propto T^{1/z}$  with  $z$  and  $\nu$  being constants called the critical exponents. All relevant quantities in this region are supposed to be universal functions  $f$  of ratio of the lengths which can be written in the form of scaling variable  $(B - B_c)/T^{1/z\nu}$ . For the resistivity in two dimensions  $R_{\square}$  this dependence takes form [10]

$$R_{\square}(B, T) = R_c f[(B - B_c)/T^{1/z\nu}], \quad (1)$$

where the critical resistance  $R_c$  is a constant.

Experimentalists managed to confirm existence of this so called finite-size scaling practically in *all* cases when checking the existence of supposed SIT. Fig. 3 presents a typical example. The question is whether the possibility to depict the data by relation (1) is a cogent argument in favor of SIT.



**Fig. 3.** Scaling of the function  $R(T, B)$  for a sample  $\text{Mo}_x\text{Ge}_{1-x}$  [1]; the changes in the upper branch lie within the range of 7%.

The insulating phase which appears as the result of such SIT is rather specific; it contains pair correlations between the localized electrons as the remnant of the superconducting pairing. Such insulator is called the Bose-insulator [11] and the correlated electrons are called localized electron pairs. Of course, existence of such phase should be confirmed experimentally.

In most cases, the fan-like shape of the set of  $R(T, B = \text{const})$  curves is accompanied by the negative magnetoresistance in higher fields, on the insulating side of the SIT but far enough from the critical region (Figs. 4 and 5). This has a natural explanation. The pair correlations are done away with the strong magnetic field and this results in raising of the carrier mobility. Similar effect is well known for granular superconductors [12, 13]: when Josephson currents are absent by some reason so that the conductance is determined by one-particle tunneling between the grains then the superconductivity of the grains results in insulating behavior of the whole material. The magnetic field

destroys the superconducting gap in the grains and hence restores metallic properties of the material.

So, the localized pairs display themselves at the stage when they become decoupled and contribute to the conductance. The negative magnetoresistance serves as an indirect manifestation of the specific insulating state destroyed by the field. However, as the theory [9, 10] relates only to the vicinity of the SIT, the negative magnetoresistance happens beyond the range of its action.

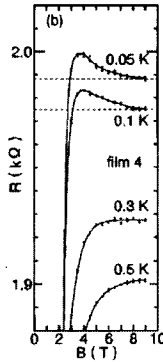


Fig. 4. Set of isotherms  $R(T=\text{const}, B)$  of  $\text{Mo}_x\text{Si}_{1-x}$  films [2].

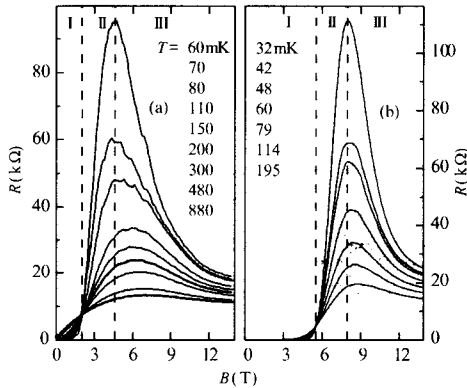


Fig. 5. Set of isotherms  $R(T=\text{const}, B)$  of amorphous  $\text{InO}_x$  films [4]. (a) Magnetic field normal to the film; (b) magnetic field parallel to the film. In the fields region I the material remains superconducting, label III marks the region of negative magnetoresistance. The theory [9, 10] relates to the vicinity of the boundary between the regions I and II in the geometry (a).

Finally, the situation looks as follows. Experiments concentrate on three specific properties as signs of the SIT: fan-like temperature dependence; scaling relation (1); negative magnetoresistance. The latter is very important

indication of the pair localization but it does not follow from the theory [9, 10]. At the same time, the data which do not display explicitly the insulating behavior can be adjusted with this theory as well. Additional questions come from the observation in amorphous  $\text{InO}_x$  films [14] that all three crucial properties of the function  $R(T, B)$  remain the same with magnetic field parallel to the film. This means that SIT is a kind of 3D-phenomenon in InO and should be explained without reference to the boson-vortex duality.

From here follow main goals in the SIT problem: to find theoretical models which would lead to the negative magnetoresistance; to trace how the specific SIT properties appear in the field-induced superconductor-normal metal transition while the normal metal is shifted toward the insulating state; to find out whether the low dimensions of the films is crucial or the SIT can happen in 3D materials; to find the explanation for the pair localization alternative to the boson-vortex duality. It seems that the first two goals are achieved.

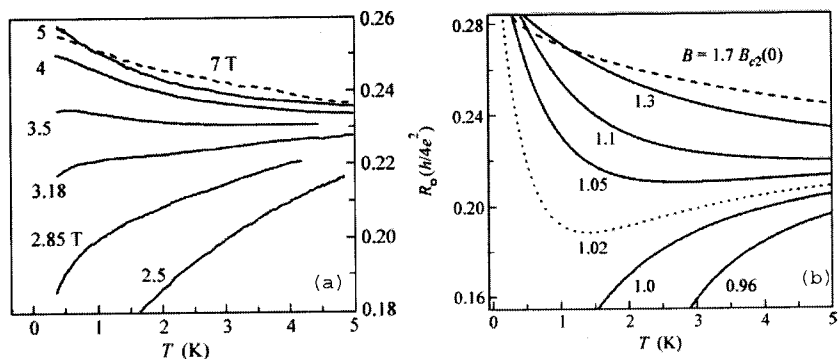
The progress came from the recent paper by Galitski and Larkin [15]. They succeeded in extending calculations of the quantum corrections due to superconducting fluctuations for 2D superconductors to the low temperature  $T \ll T_c(0)$  and high magnetic field  $B \gtrsim B_{c2}(0)$ . The corrections  $\delta\sigma$  include Aslamazov-Larkin, Maki-Thompson and density-of-states (DOS) terms. In the dirty limit  $\delta\sigma$  they can be written in explicit form by using digamma function  $\psi(r)$

$$\delta\sigma = \frac{4e^2}{3\pi h} \left[ -\ln \frac{r}{b} - \frac{3}{2r} + \psi(r) + 4(r\psi'(r) - 1) \right], \quad (2)$$

where  $r = (1/2\gamma')(b/t)$ ,  $\gamma' = e^\gamma = 1.781$  is the exponential of Euler's constant, and  $t = T/T_{c0} \ll 1$  and  $b = (B - B_{c2}(T))/B_{c2}(0) \ll 1$  are reduced temperature and magnetic field. The particular feature of this expression are the negative terms. They originate from the depression of DOS at the Fermi-level by fluctuative pairing of carriers. This becomes important if disorder and magnetic field make ineffective the transport by Cooper pairs and finally lead to the negative magnetoresistance.

Expression (2) can be compared with experiment. Fig. 6 presents such comparison made in [7]. One can see remarkable resemblance — there is separation between low-field curves which “bend down”, and high-field curves which “bend up”; there is also high field negative magnetoresistance at low temperature. Detailed analysis in [7] confirms that the fluctuations may explain all the main features of the transport in those materials where the effect is not too large and can be described in terms of the perturbation theory. In practice, almost all the materials except InO fall into this group. For them, superconducting fluctuations act as superconducting grains.

The theory [15] cannot be applied to InO directly. But just as the divergence of the weak localization quantum corrections point to the Anderson localization, quantum corrections here point to the SIT. This returns us to the problem of low dimensions. Fluctuations are larger in the systems with low dimensions, but the difference is only quantitative and we may expect SIT



**Fig. 6.** Comparison from [7] of (a) the experimental set of curves for crystalline  $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_{4+y}$  films and (b) the set calculated from the theory [15].

on the basis of fluctuations in 3D as well. These expectations are supported by the results of numerical simulations [16, 17]. According to [17], the attractive interaction leads to the insulating phase of localized pairs well within the metallic phase of single-particle 3D Anderson model.

The last comment is about the finite-size scaling equation (1) related to SIT. Certainly, expression (2) cannot be reduced to the form of equation (1) and no genuine scaling exists. However, in a restricted region of values of  $T$  and  $B$  representation of the theoretical curves in the form (1) can be done. This was demonstrated in [7]. This means that the scaling presentation cannot be the decisive argument in favor of a specific model.

*Acknowledgements.*— This work was supported by grant RFBR-02-02-16782 and by grants from Russian Ministry of Science.

## References

1. A. Yazdani and A. Kapitulnik, *Phys. Rev. Lett.* **74**, 3037 (1995).
2. S. Okuma, T. Terashima, and N. Kokubo, *Solid State Comm.* **106**, 529 (1998).
3. A.F. Hebard and M.A. Paalanen, *Phys. Rev. Lett.* **65**, 927 (1990).
4. V.F. Gantmakher, M.V. Golubkov, V.T. Dolgoplov, G.E. Tsydynzhapov, and A.A. Shashkin, *JETP Lett.* **68**, 363 (1998).
5. E. Bielejec and W. Wu, *Phys. Rev. Lett.* **88**, 206802 (2002).
6. S. Tanda, S. Ohzeki, and T. Nakayama *Phys. Rev. Lett.* **69**, 533 (1992).
7. V.F. Gantmakher, S.N. Ermolov, G.E. Tsydynzhapov, A.A. Zhukov, and T.I. Baturina. *JETP Letters* **77**, 424 (2003).
8. E. Abrahams, P.W. Anderson, D.C. Licciardello, and T.V. Ramakrishnan, *Phys. Rev. Lett.* **42**, 673 (1979).
9. S.L. Sondhi, S.M. Girvin, J.P. Carini, and D. Shahar, *Rev. Mod. Phys.* **69**, 315 (1997).
10. M.P.A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990).
11. M.A. Paalanen, A.F. Hebard, and R.R. Ruel, *Phys. Rev. Lett.* **69**, 1604 (1992).

12. V.F. Gantmakher, V.N. Zverev, V.M. Teplinskii, G.E. Tsydynhapov, and O.I. Barkalov, JETP **77**, 513 (1993).
13. I.S. Beloborodov and K.B. Efetov, Phys. Rev. B **82**, 3332 (1999).
14. V.F. Gantmakher, M.V. Golubkov, V.T. Dolgoplov, G.E. Tsydynhapov, and A.A. Shashkin. JETP Lett. **71**, 473 (2000).
15. V.M. Galitski and A.I. Larkin, Phys. Rev. B **63**, 174506 (2001).
16. J. Lages and D.L. Shepelyansky, Phys. Rev. B **64**, 094502 (2001).
17. B. Srinivasan, G. Benenti, and D.L. Shepelyansky, Phys. Rev. B **66**, 172506 (2002).