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Traits of the insulator formed under the superconductor–insulator transition

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Abstract

Various experimental observations of the magnetic-field-induced superconductor–insulator transition are described and compared with different theoretical models: one based on boson–vortex duality, next exploring the properties of granular superconductors and the third analyzing effect of the superconducting fluctuations in the magnetic field at low temperature. All the models point to the existence of pairwise electron correlations at the Fermi-level of the insulator. These so-called localized pairs should vanish in high magnetic fields. The localized pairs apparently come from the parity effect in ultrasmall quasigrains—local minima of the random potential which can admit only small limited number of electrons.

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When superconductivity is destroyed by changing some parameters, either intrinsic (carriers density, level of disorder) or extrinsic (magnetic field) ones, the material can turn not only into normal metal but into insulator as well. We'll discuss here magnetic-field-induced superconductor–insulator transitions (SIT). A system of delocalized electrons is characterized by the product of the Fermi-wavevector and mean free path $k_F l$. If the carrier density in the material is low and the level of disorder is high, so that $k_F l \approx 1$ and the material would be on the insulating side of the metal-insulating transition if it were not for the superconductivity, then the magnetic field may turn the superconductor into insulator. The main sign of SIT is the fan-like set of the resistance curves $R(T)$: they go down with decreasing of the temperature at fields below the critical, $B < B_c$, and go up at fields $B > B_c$.

The list of materials which display this kind of behavior contains amorphous Mo_xGe_{1-x} [1] and Mo_xSi_{1-x} [2] films, amorphous InO_x films [3,4], ultrathin films of Be [5], crystalline films of $Nd_{2-x}Ce_xCuO_{4+y}$ [6,7] and of TiN_x [8]. Two typical examples of such sets of curves relevant to different limits are presented in Figs. 1 and 2. In $Nd_{2-x}Ce_xCuO_{4+y}$ (Fig. 1), the growth of the resistance on the non-superconducting side of the field-induced SIT with decreasing temperature is below 10%; this reminds rather a metal with

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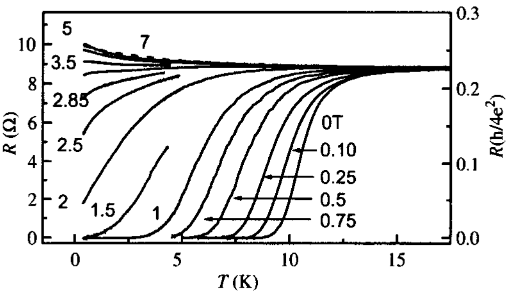

Fig. 1. Experimental set of curves for crystalline $Nd_{2-x}Ce_xCuO_{4+y}$ films [7]. At fields above B_c the resistivity changes are below 10%. Two upper curves cross at low temperatures (see enlarged part of this plot below, on Fig. 4(b)).

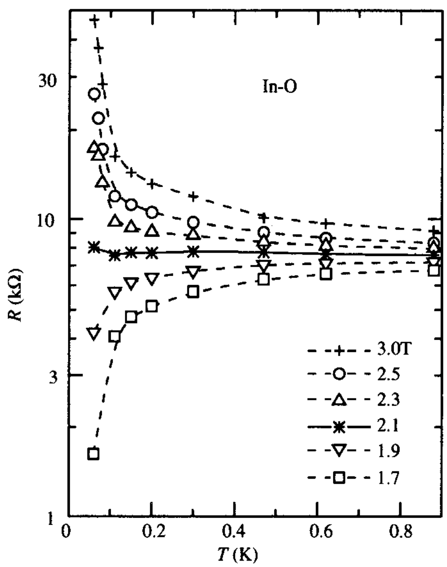

Fig. 2. Temperature dependence of the resistivity of high-resistive amorphous InO_x films at different magnetic fields [4].

quantum conductivity corrections than an insulator. Amorphous InO_x represents the opposite limit. Depending on non-stoichiometry parameter $3-x$, uprise of the low-temperature resistance may be of 50%, or may be almost 10-fold as on Fig. 2, or may even reach more than five orders of magnitude in the temperature interval 0.07–1 K [9] with typical for an insulator exponential temperature dependence of the resistance. In all cases, there exists a critical value of the magnetic field B_c at which the resistance does not depend or almost does not depend on temperature, $R(T) \approx \text{const} = R_c$.

All the transport data in the above-listed experiments were interpreted as SIT in two-dimensional (2D) electron systems. For the systems with comparatively trifling growth of the resistance ($Mo_{1-x}Ge_{1-x}$, Mo_xSi_{1-x}, $Nd_{2-x}Ce_xCuO_{4+y}$), this interpretation is based on scaling hypothesis [10] which asserts that the metallic ground state is prohibited for 2D-systems and that only insulating and superconducting ground states are possible. Hence the film states which do not display tendency to become superconducting, i.e. those with low-temperature derivative $\partial R/\partial T < 0$, should be accepted as insulating.

The theoretical model of 2D-SIT which was suggested in [11,12] appeals to similarity of two kinds of bosons: Cooper pairs and vortices, called the boson–vortex duality. The model considers the insulating phase as a condensate of vortices with localized Cooper pairs just as the superconducting phase is a condensate of Cooper pairs with localized vortices. The fan-like set of the resistance curves $R(T)$ is an essential yield of this theory. The insulating phase which appears as the result of such SIT is rather specific; it contains pairwise correlations between the localized electrons as the remnant of the superconducting pairing. It is called the Bose-insulator [13] and the correlated electrons are referred as localized electron pairs. Of course, existence of such specific insulator should be confirmed experimentally. Though the theory [11,12] describes only vicinity of the SIT and studies transport only inside the critical region on the (T, B)-plane, the confirmation came from high field region, well above the critical field B_c.

In most cases, the fan-like shape of the set of $R(T, B = \text{const})$ curves is accompanied by the negative magnetoresistance (NMr) in higher fields on the insulating side of the SIT, far away from the critical region $|B - B_c| \ll B_c$. In Fig. 1, the NMr looks as crossing of two upper curves at low temperatures. It is demonstrated in detail in Fig. 3(a) and (b) for InO_x films with higher resistance. Noteworthy that total reduction of the resistance in high fields is always of the same order of magnitude as its initial uprise above R_c.

The NMr has been studied and explained both experimentally [15] and theoretically [16] for granular superconductors. Superconducting gap in

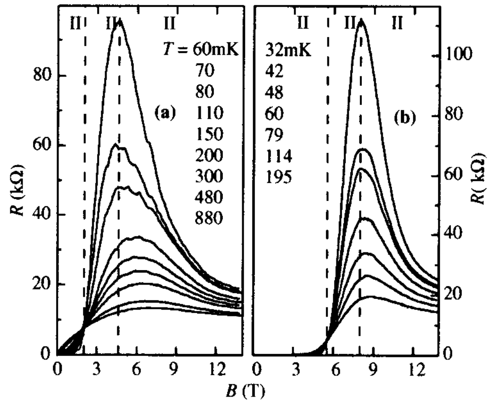

Fig. 3. Set of isotherms $R(T = \text{const}, B)$ for amorphous InO_x films [14]. In the fields region I the material remains superconducting, label III marks the NMr region. (a) Magnetic field normal to the film. The theory [11,12] relates to the vicinity of the boundary between the regions I and II. (b) Magnetic field parallel to the film (see discussion at the end of the paper).

the spectrum of grains leads to exponentially small number of excitations at low temperatures inside the grains, and hence to exponentially small one-particle intergrain tunneling current. When this tunneling process controls the total resistance, the granular material behaves as insulator. The magnetic field destroys the gap, increases one-particle intragrain density of states at the Fermi-level and reduces the tunneling resistance and hence the resistance of the granular material as a whole.

There is intrinsic similarity between the basic concepts of the scaling theory for Bose-insulator [11,12] and of the theory for granular superconductor [16]: pairs localized in the disordered materials and Cooper pairs quasilocalized inside grains; the strong magnetic field destroys pairwise correlations in the Bose-insulator and the superconductivity inside the grains in the granular insulator. But neither theory can be applied directly to NMr in homogeneously disordered materials. The theory [11,12] relates only to the vicinity of the SIT, the NMr happens beyond the range of its action. And expansion of the theory developed for granular materials to the homogeneously disordered ones is far from obvious.

The progress came from the recent paper by Galitski and Larkin [17]. They succeeded in extending calculations of the superconducting fluctuation corrections $\delta\sigma$ to the conductivity of 2D superconductors to the low temperature $T \ll T_c(0)$ and the high magnetic field $B \gtrsim B_{c2}(0)$. The correction $\delta\sigma$ includes Aslamazov–Larkin, Maki–Thompson and density-of-states (DOS) terms. In the dirty limit it can be written in the form

$$\delta\sigma = \frac{4e^2}{3\pi h}\left[-\ln\frac{r}{b} - \frac{3}{2r} + \psi(r) + 4(r\psi'(r) - 1)\right], \tag{1}$$

where ψ is digamma-function, $r = (1/2\gamma')(b/t)$, $\gamma' = e^{\gamma} = 1.781$ is the exponential of Euler's constant, and $t = T/T_{c0} \ll 1$ and $b = (B - B_{c2}(T))/B_{c2}(0) \ll 1$ are reduced temperature and magnetic field. The particular feature of this expression are the negative terms. They originate from the depression of DOS g near the Fermi-level by fluctuative pairing of carriers. This becomes important if disorder and magnetic field make ineffective the transport by Cooper pairs. The remaining transport depends on the DOS. The magnetic field oppresses the pair correlations and increases the DOS at the Fermi-level; this leads to growth of the conductance, i.e. to the NMr.

To compare the expression (1) with experiment we shall revisit Fig. 1 and present in Fig. 4(b) the low-temperature part of the set of $R(B)$ curves for $Nd_{2-x}Ce_xCuO_{4+y}$. The curves $R(B)$ measured at 2.85 and 3.18 T both show at least tendency to the superconducting behavior at the lowest available temperature. The curves at 3.5 T and even at 4 T may be a maximum at lower temperature but the curve at 5 T certainly not. So we have the sequence: *the superconducting state* ($B < 3.5 - 4$ T)—*the non-superconducting state* ($B = 5$ T)—*the state with NMr* ($B = 7$ T). Uncertainty in the critical field B_c is not of vital importance.

To make the theoretical expression suitable for the comparison, Aronov–Altshuler quantum correction $\delta_{AA}\sigma = -(e^2/h)\ln(T/T^*)$ from the diffusive channel typical for normal metals was added [7] to the formula (1):

$$\sigma = \sigma_0 + \delta\sigma(B,T) - \alpha\frac{e^2}{h}\ln(T/T^*) \tag{2}$$

(here α and T^* are adjustable parameters; allowing for $\alpha \neq 1$ we make flexible the relation between the

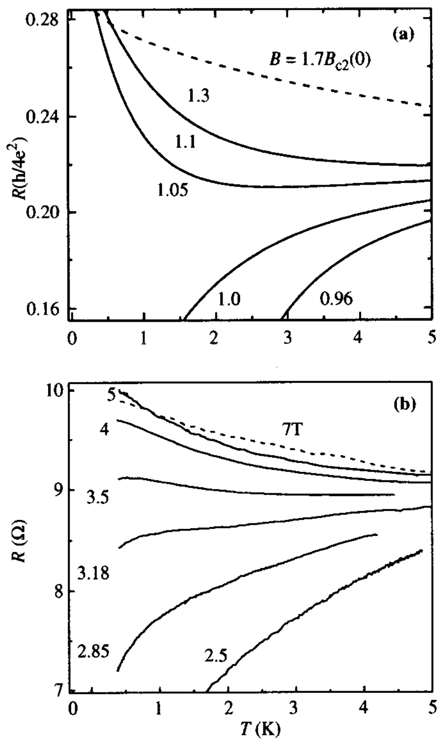

Fig. 4. Comparison of (a) the set of curves derived from the theory [17] and (b) the experimental set for crystalline $Nd_{2-x}Ce_xCuO_{4+y}$ films. Panel (b) is an enlarged part of the plot from Fig. 1 (from [7]).

contributions from Cooper and diffusive channels; the latter is assumed to be field independent). One can see in Fig. 4 remarkable resemblance in qualitatively distinguishable features of panels (a) and (b)—there is separation between low field curves which "bend down", and high field which "bend up"; there is also high field NMr at low temperature.

So, the SIT experiments are approached from different sides by three theoretical models based on the boson–vortex duality [11,12], on granularity [16], and on superconducting fluctuations [17]. All three lead to similar conclusion: the specific insulator with pairwise correlations near the Fermi-level does exist.

Superconducting fluctuations may stimulate the electron localization. This can be demonstrated by the same trick as that used by Larkin and Khmel'nitskii [18] to determine the position of the crossover from logarithmic to exponential temperature dependence in non-superconducting 2D electron gas, with only diffusive channel being important. The crossover temperature T_{00} was estimated [18] from the relation $\sigma_0 \approx (e^2/h)\ln T_{00}$ as

$$T_{00} \simeq \frac{\varepsilon_F}{k_F l} e^{-2(k_F l)}, \tag{3}$$

where ε_F was the Fermi energy [18]. Acting in the same manner and equating the right part of the relation (2) to zero, one gets temperature T_0 of the crossover to bosonic insulator state as the function of the magnetic field. The values of T_0 near maxima turn to be much larger then T_{00} (Fig. 5).

Similar results were obtained by numerical simulations [19,20]. It was demonstrated that the attractive interaction stimulated localization by combining single particles into pairs. This effect happened in 3D as well [20]. This was the second indication that SIT could exist in 3D materials; the first one followed from the model of superconducting grains. The possible dimensionality of the Bose-insulator is the next request to the experimenters.

All experiments [1–8] were done with films. It is usually difficult to say whether a film is thick enough to be considered as 3D. For instance, the

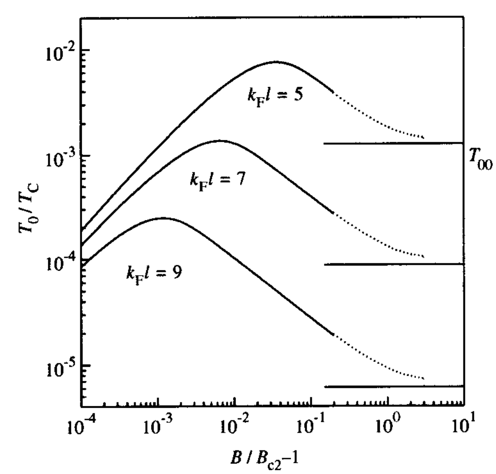

Fig. 5. Crossover temperature T_0 for several reduced values of the mean free path l calculated by equating to zero the right part of the Eq. (2) for the fields values up to $B = 1.2B_{c2}(0)$. Dotted lines qualitatively designate the asymptotic parts of the curves. Horizontal lines approximately mark levels of T_{00} derived for the same values of $k_F l$ [7].

thickness of 200 Å of InO_x films in experiments [4,14] was much larger than the mean free path $l \approx 10$ Å, but apparently slightly less than the superconducting coherent length ξ. The model based on the boson–vortex duality can be applied only to 2D-systems. But just necessity of this duality can be checked in experiment. Revisiting Fig. 3 one can see that the behavior of isotherms $R(B)$ is qualitatively the same with the field normal and parallel to the film. But in parallel field the vortices cannot be treated as particles and the duality principle does not work. Hence NMr and pairwise correlations of localized electrons have more general origin.

Qualitative model of the 3D localized pairs can be proposed when exploring the low-size limit of the granular material. A grain of size a has level spacing $\delta\varepsilon \approx (ga^3)^{-1}$; it retains superconductivity until $\delta\varepsilon$ remains less then the superconducting gap Δ_{sc}. In the opposite limit of ultrasmall grains $\delta\varepsilon \gg \Delta_{sc}$, the superconductivity is replaced by the parity effect. For an isolated grain this effect was calculated in [21]. A level in the grain occupied by two electrons has energy less by

$$\Delta_p \approx \delta\varepsilon/\ln(\delta\varepsilon/\Delta_{sc}) \quad (\delta\varepsilon \gg \Delta_{sc}), \tag{4}$$

than when it is occupied only by one electron. This would mean a gap Δ_p in one-particle spectrum for the multitude of such grains: when hopping from a double-occupied level, the electron has to provide the left one with energy Δ_p. Sure, this is only a rough notion but it can be used as a starting point in describing homogeneously disordered insulator with pairwise electron correlations.

To summarize, field and temperature dependence of superconducting fluctuations describe negative magnetoresistance of 2D-materials with moderate resistivity at fields above the critical field B_c; these fluctuations may stimulate localization in the magnetic field. This supports the model of pair localization in the process of superconductor–insulator transition in high-resistive materials which follows from the idea of 2e-boson–vortex duality. Granularity of the superconductors leads to the same transport phenomena: to the superconductor–insulator transition and to the negative magnetoresistance; hence granular and homogeneously disordered high-resistive materials are hardly distinguishable. Pair localization apparently can be derived from the parity effect in ultrasmall quasigrains of a 3D disordered material. This would result in the gap at the Fermi-level in the spectrum of one-particle excitations.

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